

BRILLIANT[®] TUTORIALS

PAPER-1: CHEMISTRY, PHYSICS & MATHEMATICS

Code:

D

SOLUTIONS TO AIEEE - 2010

Time: 3 hours

Maximum Marks: 432

IMPORTANT INSTRUCTIONS:

1. The Test Booklet consists of **90** questions. The maximum marks are **432**.
2. There are **three** parts in the question paper. The distribution of marks subject wise in each part is as under for each correct response.
Part A – Chemistry (144 marks) – Questions No. 4 to 9 and 13 to 30 consist of **FOUR (4)** marks each and Questions No. 1 to 3 and 10 to 12 consist of **EIGHT (8)** marks each for each correct response.
Part B – Physics (144 marks) – Questions No. 33 to 49 and 54 to 60 consist of **FOUR (4)** marks each and Questions No. 31 to 32 and 50 to 53 consist of **EIGHT (8)** marks each for each correct response.
Part C – Mathematics (144 marks) – Questions No. 61 to 69, 73 to 81 and 85 to 90 consist of **FOUR (4)** marks each and Questions No. 70 to 72 and 82 to 84 consist of **EIGHT (8)** marks each for each correct response.
3. Candidates will be awarded marks as stated above in Instruction No. 2 for correct response of each question. $\frac{1}{4}$ (one-fourth) marks will be deducted for indicating incorrect response of each question. **No deduction** from the total score will be made **if no response** is indicated for an item in the Answer Sheet.
4. Use **Blue/Black Ball Point Pen only** for writing particulars/markings responses on **Side-1** and **Side-2** of the Answer Sheet. **Use of pencil is strictly prohibited.**
5. No candidate is allowed to carry any textual material, printed or written, bits of papers, pager, mobile phone, any electronic device, etc., except the Admit Card inside the examination hall/room.
6. Rough work is to be done on the space provided for this purpose in the Test Booklet only. This space is given at the bottom of each page.

Name of the Candidate (in Capital letters): _____

Roll Number: in figures

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Examination Centre Number:

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Name of Examination Centre (in Capital letters): _____

Candidate's Signature: _____

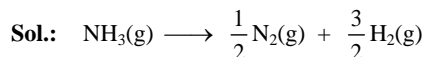
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SOLUTIONS TO AIEEE 2010
CHEMISTRY: (Code: D)
PART – A

Note: Questions with (*) mark are from syllabus of class XI.

*1. The standard enthalpy of formation of NH_3 is $-46.0 \text{ kJ mol}^{-1}$. If the enthalpy of formation of H_2 from its atoms is -436 kJ mol^{-1} and that of N_2 is -712 kJ mol^{-1} , the average bond enthalpy of N–H bond in NH_3 is

- (1) -964 kJ mol^{-1} (2) $+352 \text{ kJ mol}^{-1}$
(3) $+1056 \text{ kJ mol}^{-1}$ (4) $-1102 \text{ kJ mol}^{-1}$



$$\Delta H^\circ = -\Delta H_f^\circ(\text{NH}_3) = -(-46) = 46 \text{ kJ mol}^{-1}$$

$$\text{Also, } \Delta H^\circ = 3\Delta H_{\text{N-H}} - \frac{1}{2}\Delta H_{\text{N=N}} - \frac{3}{2}\Delta H_{\text{H-H}}$$

$$46 = 3\Delta H_{\text{N-H}} - \frac{1}{2}(712) - \frac{3}{2}(436)$$

$$\Delta H_{\text{N-H}} = \frac{1}{3}[46 + 1010] = +352 \text{ kJ mol}^{-1}$$

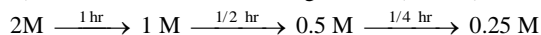
Correct choice: (2)

2. The time for half life period of a certain reaction $\text{A} \longrightarrow \text{Products}$ is 1 hour. When the initial concentration of the reactant 'A', is 2.0 mol L^{-1} , how much time does it take for its concentration to come from 0.50 to 0.25 mol L^{-1} if it is a zero order reaction?

- (1) 4 h (2) 0.5 h
(3) 0.25 h (4) 1 h

Sol.: $t_{1/2}$ for a n^{th} order reaction is given by $t_{1/2} \propto \frac{1}{[\text{A}_0]^{n-1}}$

$t_{1/2}$ for a zero order reaction is given as $t_{1/2} \propto [\text{A}_0]$



Correct choice: (3)

3. A solution containing 2.675 g of $\text{CoCl}_3 \cdot 6\text{NH}_3$ (molar mass = 267.5 g mol^{-1}) is passed through a cation exchanger. The chloride ions obtained in solution were treated with excess of AgNO_3 to give 4.78 g of AgCl (molar mass = 143.5 g mol^{-1}). The formula of the complex is (Atomic mass of Ag = 108 u)

- (1) $[\text{Co}(\text{NH}_3)_6]\text{Cl}_3$ (2) $[\text{CoCl}_2(\text{NH}_3)_4]\text{Cl}$
(3) $[\text{CoCl}_3(\text{NH}_3)_3]$ (4) $[\text{CoCl}(\text{NH}_3)_5]\text{Cl}_2$

Sol.: Number of moles of the complex = $\frac{2.675}{267.5} = 0.01$

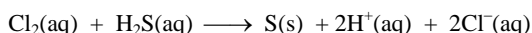
$$\text{Number of moles of AgCl formed} = \frac{4.78}{143.5} = 0.03$$

\therefore Number of moles of AgCl formed = $3 \times$ Number of moles of the complex

\therefore The formula of the complex is $[\text{Co}(\text{NH}_3)_6]\text{Cl}_3$

Correct choice: (1)

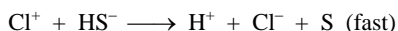
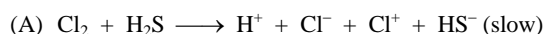
4. Consider the reaction:



The rate equation for this reaction is

$$\text{Rate} = k[\text{Cl}_2][\text{H}_2\text{S}]$$

Which of these mechanisms is/are consistent with this rate equation?

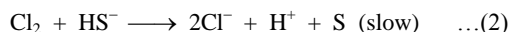
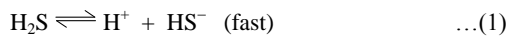


- (1) (B) only (2) Both (A) and (B)
(3) Neither (A) nor (B) (4) (A) only

Sol.: According to mechanism (A) the rate law expression is

$$\text{Rate} = k[\text{Cl}_2][\text{H}_2\text{S}]$$

According to mechanism (B)



$$\text{Rate} = k[\text{Cl}_2][\text{HS}^-]$$

From (1), $K = \frac{[\text{H}^+][\text{HS}^-]}{[\text{H}_2\text{S}]}$, or $[\text{HS}^-] = \frac{K[\text{H}_2\text{S}]}{[\text{H}^+]}$ substituting this value in rate equation.

$$\therefore \text{Rate} = k[\text{Cl}_2] K \frac{[\text{H}_2\text{S}]}{[\text{H}^+]} = k' \frac{[\text{Cl}_2][\text{H}_2\text{S}]}{[\text{H}^+]}$$

Correct choice: (4)

5. If 10^{-4} dm^3 of water is introduced into 1.0 dm^3 flask at 300 K, how many moles of water are in the vapour phase when equilibrium is established? (Given: Vapour pressure of H_2O at 300 K is 3170 Pa ; $R = 8.314 \text{ JK}^{-1} \text{ mol}^{-1}$)

(1) $5.56 \times 10^{-3} \text{ mol}$

(2) $1.53 \times 10^{-2} \text{ mol}$

(3) $4.46 \times 10^{-2} \text{ mol}$

(4) $1.27 \times 10^{-3} \text{ mol}$

Sol.: The volume occupied by water molecules in vapour phase is $(1 - 10^{-4}) \text{ dm}^3$, that is approximately 1 dm^3 .

$$P_{\text{vap}} V = n_{\text{H}_2\text{O}} RT$$

$$3170(\text{Pa}) \times 1 \times 10^{-3} (\text{m}^3) = n_{\text{H}_2\text{O}} (\text{mol}) \times 8.314 (\text{J K}^{-1} \text{ mol}^{-1}) \times 300 (\text{K})$$

$$n_{\text{H}_2\text{O}} = \frac{3170 \times 10^{-3}}{8.314 \times 300} = 1.27 \times 10^{-3}$$

Correct choice: (4)

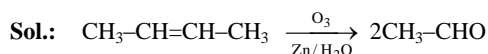
*6. One mole of a symmetrical alkene on ozonolysis gives two moles of an aldehyde having a molecular mass of 44 u. The alkene is

(1) propene

(2) 1-butene

(3) 2-butene

(4) ethene



Molecular mass of CH_3CHO is 44 u.

Correct choice: (3)

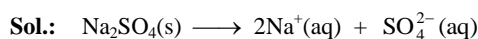
7. If sodium sulphate is considered to be completely dissociated into cations and anions in aqueous solution, the change in freezing point of water (ΔT_f), when 0.01 mol of sodium sulphate is dissolved in 1 kg of water, is ($K_f = 1.86 \text{ K kg mol}^{-1}$)

(1) 0.0372 K

(2) 0.0558 K

(3) 0.0744 K

(4) 0.0186 K



$$i = \frac{3}{1} = 3$$

$$\Delta T_f = i \cdot K_f \cdot m$$

$$= 3 \times 1.86 \times 0.01 = 0.0558 \text{ K.}$$

Correct choice: (2)

8. From amongst the following alcohols the one that would react fastest with conc. HCl and anhydrous ZnCl_2 , is

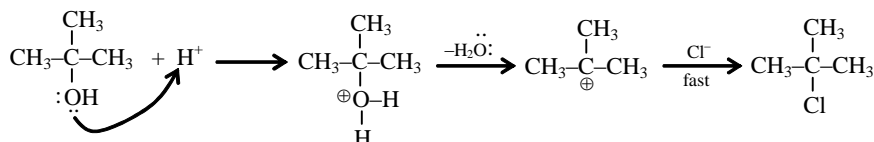
(1) 2-Butanol

(2) 2-Methylpropan-2-ol

(3) 2-Methylpropanol

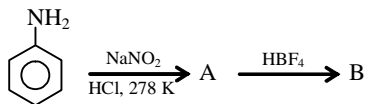
(4) 1-Butanol

Sol.: 2-Methylpropan-2-ol reacts fastest with HCl and anhydrous ZnCl_2 as it proceeds through the formation of stable tertiary carbocation.



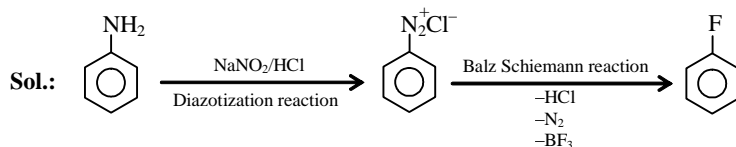
Correct choice: (2)

9. In the chemical reactions,



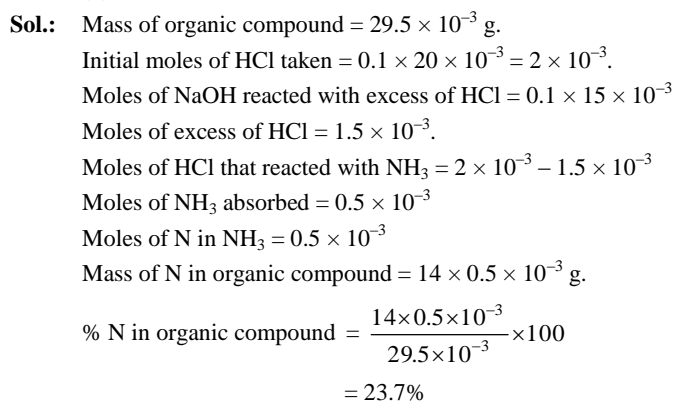
The compounds 'A' and 'B' respectively are

- (1) nitrobenzene and fluorobenzene. (2) phenol and benzene.
(3) benzene diazonium chloride and fluorobenzene. (4) nitrobenzene and chlorobenzene.



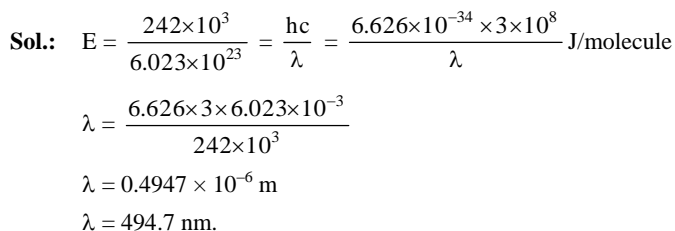
Correct choice: (3)

- *10. 29.5 mg of an organic compound containing nitrogen was digested according to Kjeldahl's method and the evolved ammonia was absorbed in 20 mL of 0.1 M HCl solution. The excess of the acid required 15 mL of 0.1 M NaOH solution for complete neutralization. The percentage of nitrogen in the compound is
- (1) 59.0 (2) 47.4
(3) 23.7 (4) 29.5



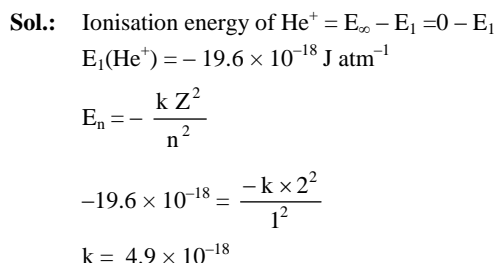
Correct choice: (3)

- *11. The energy required to break one mole of Cl-Cl bonds in Cl_2 is 242 kJ mol^{-1} . The longest wavelength of light capable of breaking a single Cl-Cl bond is ($c = 3 \times 10^8 \text{ ms}^{-1}$ and $N_A = 6.02 \times 10^{23} \text{ mol}^{-1}$)
- (1) 594 nm (2) 640 nm
(3) 700 nm (4) 494 nm



Correct choice: (4)

- *12. Ionisation energy of He^+ is $19.6 \times 10^{-18} \text{ J atom}^{-1}$. The energy of the first stationary state ($n = 1$) of Li^{2+} is
- (1) $4.41 \times 10^{-16} \text{ J atom}^{-1}$ (2) $-4.41 \times 10^{-17} \text{ J atom}^{-1}$
(3) $-2.2 \times 10^{-15} \text{ J atom}^{-1}$ (4) $8.82 \times 10^{-17} \text{ J atom}^{-1}$

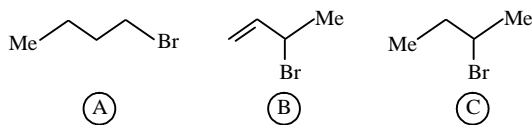


$$E_1(\text{Li}^{2+}) = -\frac{4.9 \times 10^{-18} \times 9}{1^2} = -44.1 \times 10^{-18} \text{ J atom}^{-1}$$

$$= -4.41 \times 10^{-17} \text{ J atom}^{-1}.$$

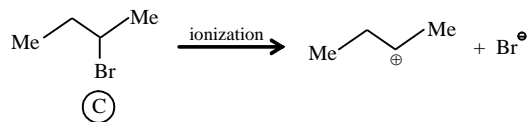
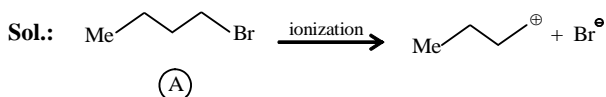
Correct choice: (2)

13. Consider the following bromides:



The correct order of S_N1 reactivity is

- (1) $B > C > A$ (2) $B > A > C$
 (3) $C > B > A$ (4) $A > B > C$



More stable the carbocation, more is the rate of S_N1 reaction. Thus, the order of S_N1 reactivity is

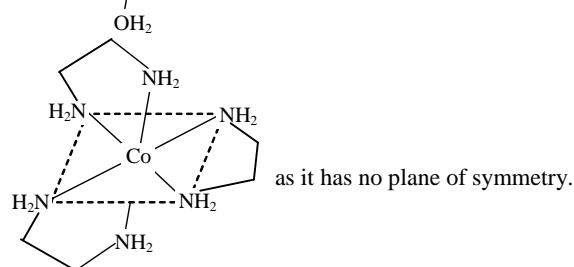
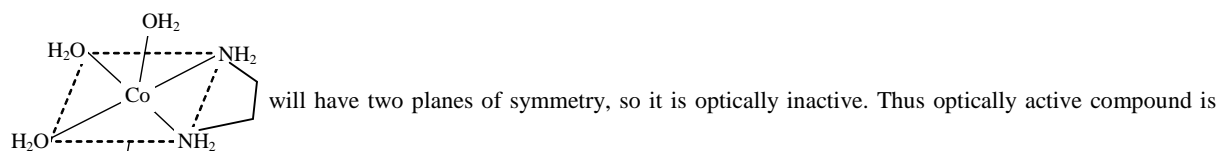
$(B) > (C) > (A)$

Correct choice: (1)

14. Which one of the following has an optical isomer?

- (1) $[\text{Zn}(\text{en})(\text{NH}_3)_2]^{2+}$ (2) $[\text{Co}(\text{en})_3]^{3+}$
 (3) $[\text{Co}(\text{H}_2\text{O})_4(\text{en})]^{3+}$ (4) $[\text{Zn}(\text{en})_2]^{2+}$
 (en = ethylenediamine)

Sol.: Square planar complexes of Zn^{2+} cannot show optical isomerism. So, options (1) and (4) are ruled out.



Correct choice: (2)

15. On mixing, heptane and octane form an ideal solution. At 373 K, the vapour pressures of the two liquid components (heptane and octane) are 105 kPa and 45 kPa respectively. Vapour pressure of the solution obtained by mixing 25.0 g of heptane and 35 g of octane will be (molar mass of heptane = 100 g mol^{-1} and of octane = 114 g mol^{-1})

- (1) 72.0 kPa (2) 36.1 kPa
 (3) 96.2 kPa (4) 144.5 kPa

The CO_3^{2-} concentration cannot be 0.034 as H_2CO_3 and HCO_3^- are weak acids. HCO_3^- dissociates very little in second step, such that $[\text{HCO}_3^-] \approx x \text{ M}$ and since $x > y$, so $[\text{HCO}_3^-] > [\text{CO}_3^{2-}]$. Concentration of $[\text{H}_3\text{O}^+] \approx x$, so $[\text{CO}_3^{2-}] \neq [\text{H}_3\text{O}^+]$. Concentration of $[\text{H}_3\text{O}^+]$ and $[\text{HCO}_3^-]$ are almost equal and its value is $x \text{ M}$.

Correct choice: (3)

19. The edge length of a face centred cubic cell of an ionic substance is 508 pm. If the radius of the cation is 110 pm, the radius of the anion is
- (1) 288 pm (2) 398 pm
(3) 618 pm (4) 144 pm

Sol.: (a) If it is a NaCl type lattice,

$$\frac{a}{2} = r_{\text{cation}} + r_{\text{anion}} \Rightarrow \frac{508}{2} = 110 + r_{\text{anion}}$$

$$r_{\text{anion}} = 144 \text{ pm}$$

(b) If it is ZnS type lattice,

$$\frac{\sqrt{3} a}{4} = r_{\text{cation}} + r_{\text{anion}}$$

$$\frac{\sqrt{3} \times 508}{4} = 110 + r_{\text{anion}}$$

$$r_{\text{anion}} = 218.2 - 110 = 108.2 \text{ pm}$$

Correct choice: (4)

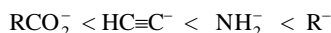
*20. The correct order of increasing basicity of the given conjugate bases ($\text{R}=\text{CH}_3$) is

- (1) $\text{RCOO}^- < \text{HC}\equiv\text{C}^- < \bar{\text{R}} < \bar{\text{N}}\text{H}_2$ (2) $\bar{\text{R}} < \text{HC}\equiv\text{C}^- < \text{RCOO}^- < \bar{\text{N}}\text{H}_2$
(3) $\text{RCOO}^- < \bar{\text{N}}\text{H}_2 < \text{HC}\equiv\text{C}^- < \bar{\text{R}}$ (4) $\text{RCOO}^- < \text{HC}\equiv\text{C}^- < \bar{\text{N}}\text{H}_2 < \bar{\text{R}}$

Sol.: The order of acidity among RCO_2H , $\text{HC}\equiv\text{CH}$, RH and NH_3 is



The order of basicity of their conjugate bases is

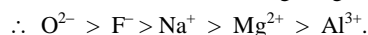


Correct choice: (4)

*21. The correct sequence which shows decreasing order of the ionic radii of the elements is

- (1) $\text{Al}^{3+} > \text{Mg}^{2+} > \text{Na}^+ > \text{F}^- > \text{O}^{2-}$ (2) $\text{Na}^+ > \text{Mg}^{2+} > \text{Al}^{3+} > \text{O}^{2-} > \text{F}^-$
(3) $\text{Na}^+ > \text{F}^- > \text{Mg}^{2+} > \text{O}^{2-} > \text{Al}^{3+}$ (4) $\text{O}^{2-} > \text{F}^- > \text{Na}^+ > \text{Mg}^{2+} > \text{Al}^{3+}$

Sol.: O^{2-} , F^- , Na^+ , Mg^{2+} and Al^{3+} are isoelectronic species having 10 electrons each. Cation having high positive charge is smallest in size and the anion with high negative charge is largest in size.



Correct choice: (4)

*22. Solubility product of silver bromide is 5.0×10^{-13} . The quantity of potassium bromide (molar mass taken as 120 g mol^{-1}) to be added to 1 litre of 0.05 M solution of silver nitrate to start the precipitation of AgBr is

- (1) $1.2 \times 10^{-10} \text{ g}$ (2) $1.2 \times 10^{-9} \text{ g}$
(3) $6.2 \times 10^{-5} \text{ g}$ (4) $5.0 \times 10^{-8} \text{ g}$

Sol.: K_{sp} of $\text{AgBr} = [\text{Ag}^+][\text{Br}^-]$

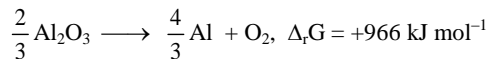
$$[\text{Br}^-] \text{ needed to precipitate AgBr} = \frac{5 \times 10^{-13}}{5 \times 10^{-2}} = 10^{-11} \text{ M}.$$

$$\text{Moles of Br}^- \text{ needed from KBr} = 10^{-11}.$$

$$\text{Mass of KBr required} = 10^{-11} \times 120 = 1.2 \times 10^{-9} \text{ g}.$$

Correct choice: (2)

23. The Gibbs energy for the decomposition of Al_2O_3 at 500°C is as follows:



The potential difference needed for electrolytic reduction of Al_2O_3 at 500°C is at least

- (1) 4.5 V (2) 3.0 V
(3) 2.5 V (4) 5.0 V

Sol.: $\Delta_r G$ given in the problem is not $+966 \text{ kJ mol}^{-1}$, it must be $+966 \text{ kJ}$.

$$\Delta_r G = +966 \times \frac{3}{2} \text{ kJ mol}^{-1}$$

$$966 \times \frac{3}{2} \times 10^3 = 6 \times 96500 \times E_{\text{cell}} \quad [\Delta G = nF E_{\text{cell}} \text{ for electrolytic cell}]$$

$$\frac{10 \times 3}{2 \times 6} = E_{\text{cell}} \quad ; \quad E_{\text{cell}} = \frac{10}{4} = 2.5 \text{ V}$$

\therefore The applied potential is 2.5 V for the electrolytic reduction of Al_2O_3 at 500°C .

Correct choice: (3)

*24. At 25°C , the solubility product of $\text{Mg}(\text{OH})_2$ is 1.0×10^{-11} . At which pH, will Mg^{2+} ions start precipitating in the form of $\text{Mg}(\text{OH})_2$ from a solution of 0.001 M Mg^{2+} ions?

- (1) 9 (2) 10
(3) 11 (4) 8

Sol.: K_{sp} of $\text{Mg}(\text{OH})_2 = [\text{Mg}^{2+}][\text{OH}^-]^2$

$$[\text{OH}^-] = \sqrt{\frac{1 \times 10^{-11}}{10^{-3}}} = \sqrt{10^{-8}} = 10^{-4} \text{ M}$$

$$\text{pOH} = 4$$

$$\therefore \text{pH} = 10$$

Correct choice: (2)

25. Percentages of free space in cubic close packed structure and in body centered packed structure are respectively

- (1) 30% and 26% (2) 26% and 32%
(3) 32% and 48% (4) 48% and 26%

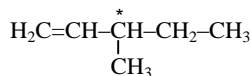
Sol.: Packing efficiency of CCP and BCC structures are 74% and 68% respectively, so the % of free space in CCP and BCC are 26% and 32% respectively.

Correct choice: (2)

26. Out of the following, the alkene that exhibits optical isomerism is

- (1) 3-methyl-2-pentene (2) 4-methyl-1-pentene
(3) 3-methyl-1-pentene (4) 2-methyl-2-pentene

Sol.: Only 3-methyl-1-pentene has a chiral carbon atom, responsible for its optical rotation.



Correct choice: (3)

27. Biuret test is *not* given by

- (1) carbohydrates (2) polypeptides
(3) urea (4) proteins

Sol.: Polypeptides, proteins and urea have $\text{-}\overset{\text{O}}{\parallel}{\text{C}}\text{-NH-}$ (peptide) linkage while carbohydrates have glycosidic linkages. So, test of

carbohydrates should be different from that of other three. Biuret test produces violet colour on addition of dilute CuSO_4 to alkaline solution of a compound containing peptide linkage.

Correct choice: (1)

28. The correct order of $E_{\text{M}^{2+}/\text{M}}^\circ$ values with negative sign for the four successive elements Cr, Mn, Fe and Co is

- (1) $\text{Mn} > \text{Cr} > \text{Fe} > \text{Co}$ (2) $\text{Cr} > \text{Fe} > \text{Mn} > \text{Co}$
(3) $\text{Fe} > \text{Mn} > \text{Cr} > \text{Co}$ (4) $\text{Cr} > \text{Mn} > \text{Fe} > \text{Co}$

Sol.: The value of $E_{M^{2+}/M}^{\circ}$ for given metal ions are $E_{Mn^{2+}/Mn}^{\circ} = -1.18$ V, $E_{Cr^{2+}/Cr}^{\circ} = -0.9$ V, $E_{Fe^{2+}/Fe}^{\circ} = -0.44$ V and $E_{Co^{2+}/Co}^{\circ} = -0.28$ V.

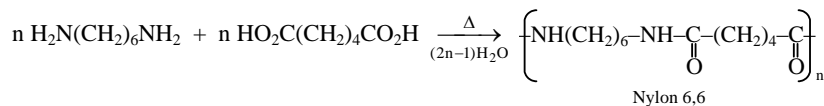
The correct order of $E_{M^{2+}/M}^{\circ}$ values *without considering negative sign would be* $Mn^{2+} > Cr^{2+} > Fe^{2+} > Co^{2+}$.

Correct choice: (1)

29. The polymer containing strong intermolecular forces e.g. hydrogen bonding, is

- (1) teflon (2) nylon 6,6
(3) polystyrene (4) natural rubber

Sol.: Nylon 6,6 has amide linkages capable of forming hydrogen bonding.



Correct choice: (2)

***30.** For a particular reversible reaction at temperature T, ΔH and ΔS were found to be both +ve. If T_e is the temperature at equilibrium, the reaction would be spontaneous when

- (1) $T_e > T$ (2) $T > T_e$
(3) T_e is 5 times T (4) $T = T_e$

Sol.: At equilibrium, $\Delta G = \Delta H - T_e \Delta S = 0$

$$T_e = \frac{\Delta H}{\Delta S}$$

For a spontaneous reaction, ΔG must be negative.

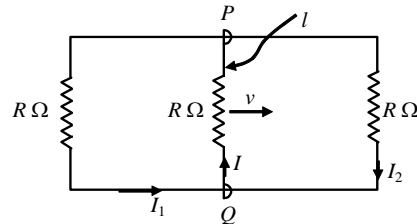
This implies $\Delta H < T \Delta S$

$$\frac{\Delta H}{\Delta S} < T ; T_e < T ; T > T_e$$

Correct choice: (2)

**SOLUTIONS TO AIEEE 2010
PHYSICS (Code: D)
PART - B**

31. A rectangular loop has a sliding connector PQ of length l and resistance $R\Omega$ and it is moving with a speed v as shown. The set-up is placed in a uniform magnetic field going into the plane of the paper. The three currents I_1 , I_2 and I are



$$(1) I_1 = -I_2 = \frac{Blv}{R}, I = \frac{2Blv}{R}$$

$$(2) I_1 = I_2 = \frac{Blv}{3R}, I = \frac{2Blv}{3R}$$

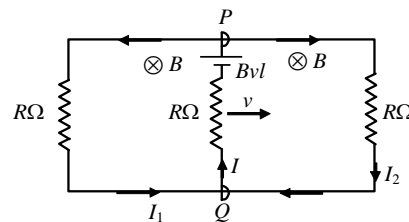
$$(3) I_1 = I_2 = I = \frac{Blv}{R}$$

$$(4) I_1 = I_2 = \frac{Blv}{6R}, I = \frac{Blv}{3R}$$

- Sol.:** Emf induced across $PQ = Blv$ and the equivalent circuit becomes as shown in the figure

$$R_{eq} = \frac{R}{2} + R = \frac{3R}{2}$$

$$\therefore I = \frac{2Blv}{3R} \text{ and } I_1 = I_2 = \frac{Blv}{3R}$$



Correct choice: (2)

32. Let C be the capacitance of a capacitor discharging through a resistor R . Suppose t_1 is the time taken for the energy stored in the capacitor to reduce to half its initial value and t_2 is the time taken for the charge to reduce to one-fourth its initial value. Then the ratio t_1/t_2 will be

$$(1) 1$$

$$(2) \frac{1}{2}$$

$$(3) \frac{1}{4}$$

$$(4) 2$$

- Sol.:** As for discharging RC circuit $q = Q_0 e^{-t/RC}$ and $U = U_0 e^{-2t/RC}$

$$\Rightarrow \frac{U_0}{2} = U_0 e^{-2t_1/RC} \text{ and } \frac{Q_0}{4} = Q_0 e^{-t_2/RC}$$

$$\text{Solving these we get } \frac{t_1}{t_2} = \frac{1}{4}$$

Correct choice: (3)

Directions: Questions number 33-34 contain Statement-1 and Statement-2. Of the four choices given after the statements, choose the one that best describes the two statements.

- *33. **Statement-1:** Two particles moving in the same direction do not lose all their energy in a completely inelastic collision.
Statement-2: Principle of conservation of momentum holds true for all kinds of collisions.
- (1) Statement -1 is true, Statement- 2 is true; Statement -2 is the correct explanation of Statement-1.
(2) Statement -1 is true, Statement- 2 is true; Statement -2 is **not** the correct explanation of Statement-1.
(3) Statement -1 is false, Statement- 2 is true
(4) Statement -1 is true, Statement- 2 is false

- Sol.:** By conservation of momentum, magnitude of momentum can not become zero.
So kinetic energy can not be lost completely

Correct choice: (1)

34. **Statement-1:** When ultraviolet light is incident on a photocell, its stopping potential is V_0 and the maximum kinetic energy of the photoelectrons is K_{\max} . When the ultraviolet light is replaced by X-rays, both V_0 and K_{\max} increase.

Statement-2: Photoelectrons are emitted with speeds ranging from zero to a maximum value because of the range of frequencies present in the incident light.

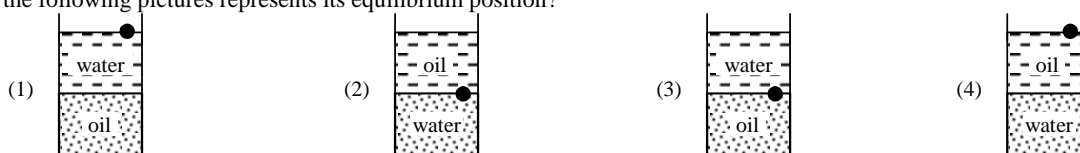
- (1) Statement -1 is true, Statement- 2 is true; Statement -2 is the correct explanation of Statement-1.
 (2) Statement -1 is true, Statement- 2 is true; Statement -2 is **not** the correct explanation of Statement-1.
 (3) Statement -1 is false, Statement- 2 is true
 (4) Statement -1 is true, Statement- 2 is false

Sol.: $h\nu - h\nu_0 = K_{\max} = eV_0$

$\nu_{\text{violet}} < \nu_{\text{X-rays}} \quad \therefore \quad V_0 \text{ and } K_{\max} \text{ are more for X-rays than ultraviolet rays}$

Correct choice: (4)

*35. A ball is made of a material of density ρ where $\rho_{\text{oil}} < \rho < \rho_{\text{water}}$ with ρ_{oil} and ρ_{water} representing the densities of oil and water, respectively. The oil and water are immiscible. If the above ball is in equilibrium in a mixture of this oil and water, which of the following pictures represents its equilibrium position?



Sol.: Water must be below as $\rho_{\text{oil}} < \rho_{\text{water}}$ and ball cannot float on oil because $\rho_{\text{oil}} < \rho$

Correct choice: (2)

*36. A particles is moving with velocity $\vec{v} = K(y\hat{i} + x\hat{j})$, where K is a constant. The general equation for its path is

- (1) $y = x^2 + \text{constant}$ (2) $y^2 = x + \text{constant}$ (3) $xy = \text{constant}$ (4) $y^2 = x^2 + \text{constant}$

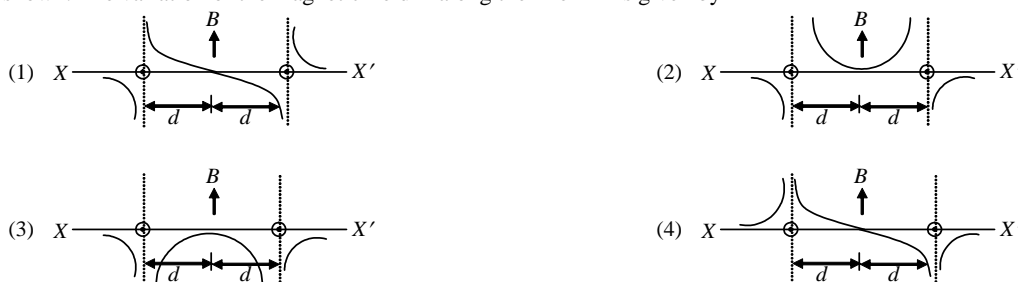
Sol.: $\vec{v} = Ky\hat{i} + Kx\hat{j}$

$\therefore \frac{dx}{dt} = Ky \text{ and } \frac{dy}{dt} = Kx \Rightarrow \frac{dy}{dx} = \frac{x}{y} \Rightarrow ydy = xdx$

$\therefore y^2 = x^2 + \text{constant}$

Correct choice: (4)

37. Two long parallel wires are at a distance $2d$ apart. They carry steady equal currents flowing out of the plane of the paper as shown. The variation of the magnetic field B along the line XX' is given by



Sol.: As the direction and magnitude of current is same in both the wires $\Rightarrow B$ is zero at mid of line joining wire 1 and 2

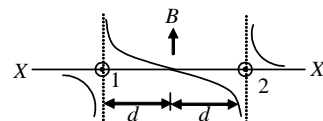
Just right of wire 1, $B > 0$

Just left of wire 1, $B < 0$

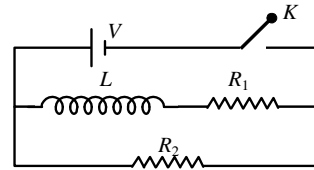
Just right of wire 2, $B > 0$

Just left of wire 2, $B < 0$

Correct choice: (1)



38. In the circuit shown, the key K is closed at $t = 0$. The current through the battery is



- (1) $\frac{VR_1R_2}{\sqrt{R_1^2 + R_2^2}}$ at $t = 0$ and $\frac{V}{R_2}$ at $t = \infty$
 (2) $\frac{V}{R_2}$ at $t = 0$ and $\frac{V(R_1 + R_2)}{R_1R_2}$ at $t = \infty$
 (3) $\frac{V}{R_2}$ at $t = 0$ and $\frac{VR_1R_2}{\sqrt{R_1^2 + R_2^2}}$ at $t = \infty$
 (4) $\frac{V(R_1 + R_2)}{R_1R_2}$ at $t = 0$ and $\frac{V}{R_2}$ at $t = \infty$

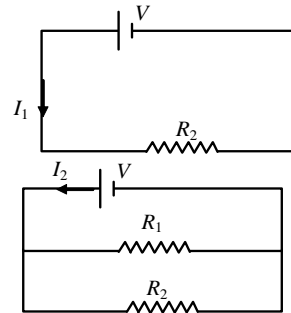
Sol.: At time $t = 0$ equivalent circuit becomes

$$I_1 = \frac{V}{R_2}$$

At time $t = \infty$

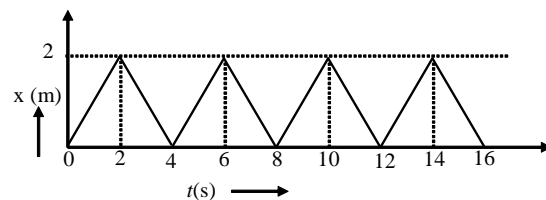
$$I_2 = \frac{V}{\frac{R_1R_2}{R_1 + R_2}} = \frac{V(R_1 + R_2)}{R_1R_2}$$

Correct choice: (2)



- *39. The figure shows the position-time ($x - t$) graph of one-dimensional motion of a body of mass 0.4 kg. The magnitude of each impulse is

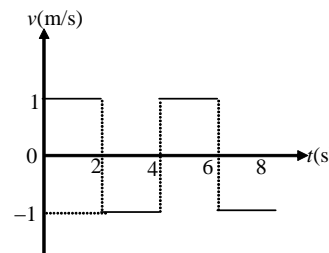
- (1) 0.4 Ns
 (2) 0.8 Ns
 (3) 1.6 Ns
 (4) 0.2 Ns



Sol.: The velocity-time ($v-t$) curve of body will be,

$$\therefore \text{Impulse} = 0.4 \times [1 - (-1)] = 0.8 \text{ Ns}$$

Correct choice: (2)



Directions: Questions number 40-41 are based on the following paragraph.

A nucleus of mass $M + \Delta m$ is at rest and decays into two daughter nuclei of equal mass $\frac{M}{2}$ each. Speed of light is c .

40. The binding energy per nucleon for the parent nucleus is E_1 and that for the daughter nuclei is E_2 . Then

- (1) $E_2 = 2E_1$ (2) $E_1 > E_2$ (3) $E_2 > E_1$ (4) $E_1 = 2E_2$

Sol.: Since, mass of daughter nuclei $<$ Mass of parent nucleus

\therefore energy is released \Rightarrow B.E per nucleon for products is greater than B.E. per nucleon for the parent nucleus

Correct choice: (3)

41. The speed of daughter nuclei is

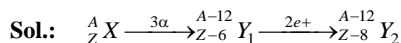
- (1) $c \frac{\Delta m}{M + \Delta m}$ (2) $c \sqrt{\frac{2\Delta m}{M}}$ (3) $c \sqrt{\frac{\Delta m}{M}}$ (4) $c \sqrt{\frac{\Delta m}{M + \Delta m}}$

Sol.: As the energy released $= (\Delta m)c^2 = \frac{1}{2} \left[\frac{M}{2} \right] v^2 \times 2 \Rightarrow v = c \sqrt{\frac{2\Delta m}{M}}$

Correct choice: (2)

42. A radioactive nucleus (initial mass number A and atomic number Z) emits 3- α particles and 2 positrons. The ratio of number of neutrons to that of protons in the final nucleus will be

(1) $\frac{A-Z-8}{Z-4}$ (2) $\frac{A-Z-4}{Z-8}$ (3) $\frac{A-Z-12}{Z-4}$ (4) $\frac{A-Z-4}{Z-2}$



Number of protons = $Z - 8$

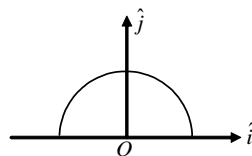
Number of neutrons = $A - 12 - (Z - 8) = A - Z - 4$

$$\therefore \text{required ratio} = \frac{A-Z-4}{Z-8}$$

Correct choice: (2)

43. A thin semi-circular ring of radius r has a positive charge q distributed uniformly over it. The net field \vec{E} at the centre O is

(1) $\frac{q}{4\pi^2 \epsilon_0 r^2} \hat{j}$ (2) $-\frac{q}{4\pi^2 \epsilon_0 r^2} \hat{j}$
(3) $-\frac{q}{2\pi^2 \epsilon_0 r^2} \hat{j}$ (4) $\frac{q}{2\pi^2 \epsilon_0 r^2} \hat{j}$



Sol.: Electric field intensity due to uniformly charged circular arc subtending angle θ_0 at its centre is

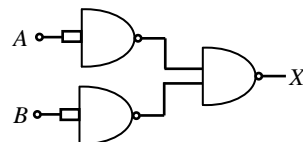
$$E = \frac{Kq}{r^2} \frac{\sin \frac{\theta_0}{2}}{\frac{\theta_0}{2}}, \text{ here } \theta_0 = \pi \text{ and } K = \frac{1}{4\pi\epsilon_0}$$

$$\therefore \vec{E} = -\frac{q}{2\pi^2 \epsilon_0 r^2} \hat{j}$$

Correct choice: (3)

44. The combination of gates shown below yields

(1) OR gate (2) NOT gate
(3) XOR gate (4) NAND gate



Sol.: $\overline{\overline{A \cdot B}} = \overline{\overline{A} \cdot \overline{B}} = A + B$

\therefore OR gate

Correct choice: (1)

- *45. A diatomic ideal gas is used in a Carnot engine as the working substance. If during the adiabatic expansion part of the cycle the volume of the gas increases from V to $32V$, the efficiency of the engine is

(1) 0.5 (2) 0.75 (3) 0.99 (4) 0.25

Sol.: $T_1 V^{\gamma-1} = T_2 (32V)^{\gamma-1}$

$$\therefore T_2 = \frac{T_1}{4} \Rightarrow \frac{T_2}{T_1} = \frac{1}{4}, \text{ as efficiency of cycle } (\eta) = 1 - \frac{T_2}{T_1} \Rightarrow \eta = 0.75$$

Correct choice: (2)

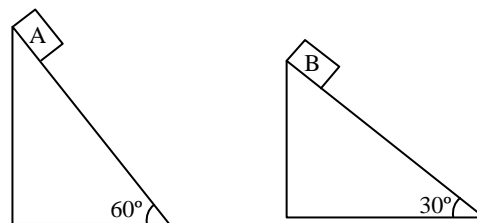
46. If a source of power 4 kW produces 10^{20} photons/second, the radiation belongs to a part of the spectrum called

(1) X-rays (2) ultraviolet rays (3) microwaves (4) γ -rays

Sol.: $P = \frac{nhc}{\lambda} \Rightarrow \lambda = \frac{nhc}{P} \Rightarrow \lambda = \frac{10^{20} \times 6.62 \times 10^{-34} \times 3 \times 10^8}{4 \times 10^3} = 4.965 \times 10^{-9} \text{ m} = 49.65 \text{ \AA}$

Correct choice: (1)

***54.** Two fixed frictionless inclined planes making an angle 30° and 60° with the vertical are shown in the figure. Two blocks A and B are placed on the two planes. What is the relative vertical acceleration of A with respect to B ?



- (1) 4.9 ms^{-2} in horizontal direction
- (2) 9.8 ms^{-2} in vertical direction
- (3) Zero
- (4) 4.9 ms^{-2} in vertical direction

Sol.: The required relative acceleration,

$$a_y = a_A \sin 60^\circ - a_B \sin 30^\circ$$

$$(a_A = g \sin 60^\circ \text{ and } a_B = g \sin 30^\circ)$$

$$\Rightarrow a_y = 4.9 \text{ m/s}^2 \text{ (in vertical direction)}$$

Correct choice: (4)

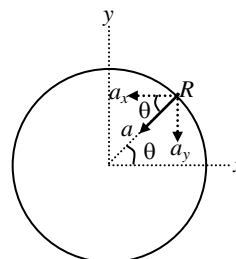
***55.** For a particle in uniform circular motion, the acceleration \vec{a} at a point $P(R, \theta)$ on the circle of radius R is (Here θ is measured from the x -axis)

- (1) $-\frac{v^2}{R} \cos \theta \hat{i} + \frac{v^2}{R} \sin \theta \hat{j}$
- (2) $-\frac{v^2}{R} \sin \theta \hat{i} + \frac{v^2}{R} \cos \theta \hat{j}$
- (3) $-\frac{v^2}{R} \cos \theta \hat{i} - \frac{v^2}{R} \sin \theta \hat{j}$
- (4) $\frac{v^2}{R} \hat{i} + \frac{v^2}{R} \hat{j}$

Sol.: $a = \frac{v^2}{r}$ and $\vec{a} = -a \cos \theta \hat{i} - a \sin \theta \hat{j}$

$$= \frac{v^2}{r} (-\cos \theta \hat{i} - \sin \theta \hat{j})$$

$$\vec{a} = -\frac{v^2}{r} \cos \theta \hat{i} - \frac{v^2}{r} \sin \theta \hat{j}$$



Correct choice: (3)

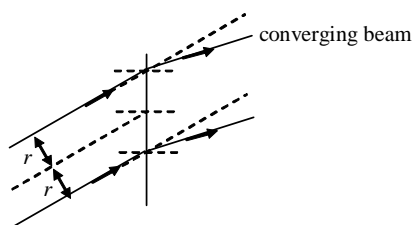
Directions: Questions number 56 – 58 are based on the following paragraph.

An initially parallel cylindrical beam travels in a medium of refractive index $\mu(I) = \mu_0 + \mu_2 I$, where μ_0 and μ_2 are positive constants and I is the intensity of the light beam. The intensity of the beam is decreasing with increasing radius.

56. As the beam enters the medium, it will

- (1) diverge
- (2) converge
- (3) diverge near the axis and converge near the periphery
- (4) travel as a cylindrical beam

Sol.:



Correct choice: (2)

57. The initial shape of the wavefront of the beam is
 (1) convex (2) concave
 (3) convex near the axis and concave near the periphery (4) planar

Sol.: Correct choice: (4)

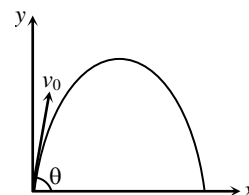
58. The speed of light in the medium is
 (1) minimum on the axis of the beam (2) the same everywhere in the beam
 (3) directly proportional to the intensity I (4) maximum on the axis of the beam

Sol.: Refractive index is maximum at axis. Therefore speed of light at axis will be minimum.

Correct choice: (1)

- *59. A small particle of mass m is projected at an angle θ with the x -axis with an initial velocity v_0 in the x - y plane as shown in the figure. At a time $t < \frac{v_0 \sin \theta}{g}$, the angular momentum of the particle is

- (1) $-mg v_0 t^2 \cos \theta \hat{j}$ (2) $mg v_0 t \cos \theta \hat{k}$
 (3) $-\frac{1}{2} mg v_0 t^2 \cos \theta \hat{k}$ (4) $\frac{1}{2} mg v_0 t^2 \cos \theta \hat{i}$



where \hat{i} , \hat{j} and \hat{k} are unit vectors along x , y and z -axis respectively.

Sol.: The position vector and velocity vector of projectile at any time t is given as

$$\vec{r}_p = (v_0 \cos \theta)t \hat{i} + \left(v_0 \sin \theta t - \frac{1}{2} g t^2 \right) \hat{j}$$

$$\vec{v}_p = v_0 \cos \theta \hat{i} + (v_0 \sin \theta - g t) \hat{j}$$

Angular momentum of the particle about origin

$$\vec{L} = m \vec{r}_p \times \vec{v}_p$$

$$\Rightarrow \vec{L} = -\frac{1}{2} mg v_0 t^2 \cos \theta \hat{k}$$

Correct choice: (3)

- *60. The equation of a wave on a string of linear mass density 0.04 kg m^{-1} is given by $y = 0.02(\text{m}) \sin \left[2\pi \left(\frac{t}{0.04(\text{s})} - \frac{x}{0.50(\text{m})} \right) \right]$.

The tension in the string is

- (1) 4.0 N (2) 12.5 N (3) 0.5 N (4) 6.25 N

Sol.: $v = \frac{\omega}{K} = 12.5 \text{ m/s} \left(\omega = \frac{2\pi}{0.04} \text{ and } K = \frac{2\pi}{0.5} \right)$

and $\mu = 0.04 \text{ kg m}^{-1}$.

Also $v = \sqrt{\frac{T}{\mu}}$

$\Rightarrow T = 6.25 \text{ N}$

Correct choice: (4)

**SOLUTIONS TO AIEEE 2010
MATHEMATICS (CODE: D)**

PART - C

*61. Let $\cos(\alpha + \beta) = \frac{4}{5}$ and let $\sin(\alpha - \beta) = \frac{5}{13}$, where $0 \leq \alpha, \beta \leq \frac{\pi}{4}$. Then $\tan 2\alpha =$

- (1) $\frac{56}{33}$ (2) $\frac{19}{12}$ (3) $\frac{20}{7}$ (4) $\frac{25}{16}$

Sol.: $\cos(\alpha + \beta) = \frac{4}{5}$, $\sin(\alpha - \beta) = \frac{5}{13}$, $0 \leq \alpha, \beta \leq \frac{\pi}{4}$

$$\tan 2\alpha = \tan(\alpha + \beta + \alpha - \beta) = \frac{\tan(\alpha + \beta) + \tan(\alpha - \beta)}{1 - \tan(\alpha + \beta)\tan(\alpha - \beta)} = \frac{\frac{3}{4} + \frac{5}{12}}{1 - \frac{3}{4} \times \frac{5}{12}} = \frac{56}{33}$$

Correct choice: (1)

*62. Let S be a non-empty subset of \mathbf{R} . Consider the following statement:

P : There is a rational number $x \in S$ such that $x > 0$.

Which of the following statements is the negation of the statement P ?

- (1) There is no rational number $x \in S$ such that $x \leq 0$.
 (2) Every rational number $x \in S$ satisfies $x \leq 0$.
 (3) $x \in S$ and $x \leq 0 \Rightarrow x$ is not rational.
 (4) There is a rational number $x \in S$ such that $x \leq 0$.

Sol.: **Correct choice: (2)**

63. Let $\vec{a} = \hat{j} - \hat{k}$ and $\vec{c} = \hat{i} - \hat{j} - \hat{k}$. Then the vector \vec{b} satisfying $\vec{a} \times \vec{b} + \vec{c} = \vec{0}$ and $\vec{a} \cdot \vec{b} = 3$ is

- (1) $2\hat{i} - \hat{j} + 2\hat{k}$ (2) $\hat{i} - \hat{j} - 2\hat{k}$ (3) $\hat{i} + \hat{j} - 2\hat{k}$ (4) $-\hat{i} + \hat{j} - 2\hat{k}$

Sol.: $\vec{a} \times \vec{b} = -\vec{c}$

$$(\vec{a} \times \vec{b}) \times \vec{a} = -\vec{c} \times \vec{a}$$

$$2\vec{b} - 3\vec{a} = \vec{a} \times \vec{c}, \text{ where } \vec{a} \times \vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 1 & -1 \\ 1 & -1 & -1 \end{vmatrix} = \hat{i}(-1-1) - \hat{j}(1) + \hat{k}(-1) = -2\hat{i} - \hat{j} - \hat{k}$$

$$2\vec{b} = -2\hat{i} - \hat{j} - \hat{k} + 3\hat{j} - 3\hat{k} \Rightarrow 2\vec{b} = -2\hat{i} + 2\hat{j} - 4\hat{k}$$

$$\Rightarrow \vec{b} = -\hat{i} + \hat{j} - 2\hat{k}$$

Correct choice: (4)

64. The equation of the tangent to the curve $y = x + \frac{4}{x^2}$, that is parallel to the x -axis, is

- (1) $y = 1$ (2) $y = 2$ (3) $y = 3$ (4) $y = 0$

Sol.: $\frac{dy}{dx} = 1 - \frac{8}{x^3} = 0 \Rightarrow x = 2$ and $y = 2 + \frac{4}{4} = 3$

Equation of tangent is $y = 3$

Correct choice: (3)

65. Solution of the differential equation $\cos x dy = y(\sin x - y) dx$, $0 < x < \frac{\pi}{2}$ is

- (1) $y \sec x = \tan x + c$ (2) $y \tan x = \sec x + c$ (3) $\tan x = (\sec x + c)y$ (4) $\sec x = (\tan x + c)y$

Sol.: $-\frac{1}{y^2} \frac{dy}{dx} + \frac{1}{y} \tan x = \sec x$

Let $\frac{1}{y} = t$; $-\frac{1}{y^2} \cdot \frac{dy}{dx} = \frac{dt}{dx}$

$\Rightarrow \frac{dt}{dx} + t \tan x = \sec x \Rightarrow \text{I.F.} = e^{\int \tan x dx} = \sec x$

$\Rightarrow t \cdot \sec x = \int \sec^2 x dx \Rightarrow \sec x = y \tan x + cy \Rightarrow \sec x = y(\tan x + c)$

Correct choice: (4)

66. The area bounded by the curves $y = \cos x$ and $y = \sin x$ between the ordinates $x = 0$ and $x = \frac{3\pi}{2}$ is

(1) $4\sqrt{2} + 2$

(2) $4\sqrt{2} - 1$

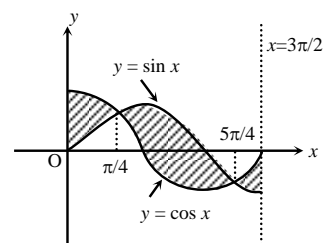
(3) $4\sqrt{2} + 1$

(4) $4\sqrt{2} - 2$

Sol.: Required area = $\int_0^{\pi/4} (\cos x - \sin x) dx + \int_{\pi/4}^{5\pi/4} (\sin x - \cos x) dx + \int_{5\pi/4}^{3\pi/2} (\cos x - \sin x) dx$

$= (\sqrt{2} - 1) + (\sqrt{2} + \sqrt{2}) + (\sqrt{2} - 1) = 4\sqrt{2} - 2$

Correct choice: (4)



***67.** If two tangents drawn from a point P to the parabola $y^2 = 4x$ are at right angles, then the locus of P is

(1) $2x + 1 = 0$

(2) $x = -1$

(3) $2x - 1 = 0$

(4) $x = 1$

Sol.: Locus of point P is directrix of parabola i.e., $x = -1$

Correct choice: (2)

68. If the vectors $\vec{a} = \hat{i} - \hat{j} + 2\hat{k}$, $\vec{b} = 2\hat{i} + 4\hat{j} + \hat{k}$ and $\vec{c} = \lambda\hat{i} + \hat{j} + \mu\hat{k}$ are mutually orthogonal, then $(\lambda, \mu) =$

(1) $(2, -3)$

(2) $(-2, 3)$

(3) $(3, -2)$

(4) $(-3, 2)$

Sol.: $\vec{a} \cdot \vec{c} = 0$ and $\vec{b} \cdot \vec{c} = 0$

$\lambda - 1 + 2\mu = 0$ and $2\lambda + 4 + \mu = 0 \Rightarrow \lambda + 2\mu = 1$ and $2\lambda + \mu = -4$

On solving $\mu = 2, \lambda = -3$

Correct choice: (4)

69. Consider the following relations:

$R = \{(x, y) \mid x, y \text{ are real numbers and } x = wy \text{ for some rational number } w\}$;

$S = \left\{ \left(\frac{m}{n}, \frac{p}{q} \right) \mid m, n, p \text{ and } q \text{ are integers such that } n, q \neq 0 \text{ and } qm = pn \right\}$.

Then

(1) neither R nor S is an equivalence relation

(2) S is an equivalence relation but R is not an equivalence relation

(3) R and S both are equivalence relations

(4) R is an equivalence relation but S is not an equivalence relation

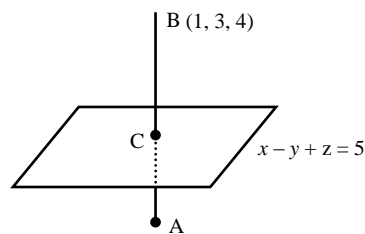
Sol.: R is not equivalence $\because (0, 2) \in R$ but $(2, 0) \notin R$

S is equivalence $\because \left(\frac{m}{n}, \frac{m}{n} \right) \in S$

$\left(\frac{m}{n}, \frac{p}{q} \right) \in S \Rightarrow \left(\frac{p}{q}, \frac{m}{n} \right) \in S$ and $\left(\frac{m}{n}, \frac{p}{q} \right), \left(\frac{p}{q}, \frac{r}{s} \right) \in S \Rightarrow \left(\frac{m}{n}, \frac{r}{s} \right) \in S$

Correct choice: (2)

Sol.: Equation of BC is $\frac{x-1}{1} = \frac{y-3}{-1} = \frac{z-4}{1} = \lambda$
 $C(\lambda+1, -\lambda+3, \lambda+4)$ as C is on the plane.
 $\therefore \lambda+1+\lambda-3+\lambda+4=5 \Rightarrow 3\lambda=3 \Rightarrow \lambda=1$
 $\Rightarrow C(2, 2, 5) \Rightarrow A(3, 1, 6)$



Statement I and II both are true but statement II is not the correct explanation.

\therefore Plane is not just bisector but it is perpendicular bisector of AB .

Correct choice: (1)

*74. Let $S_1 = \sum_{j=1}^{10} j(j-1)^{10} C_j$, $S_2 = \sum_{j=1}^{10} j^{10} C_j$ and $S_3 = \sum_{j=1}^{10} j^2 {}^{10} C_j$.

Statement-1: $S_3 = 55 \times 2^9$.

Statement-2: $S_1 = 90 \times 2^8$ and $S_2 = 10 \times 2^8$.

(1) Statement-1 is true, Statement-2 is true; Statement-2 is **not** a correct explanation for Statement-1.

(2) Statement-1 is true, Statement-2 is false.

(3) Statement-1 is false, Statement-2 is true.

(4) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

Sol.: $S_2 = \sum_{j=1}^{10} j^{10} C_j = \sum_{j=1}^{10} 10 {}^9 C_{j-1} = 10 \cdot 2^9$

Correct choice: (2)

75. Let A be a 2×2 matrix with non-zero entries and let $A^2 = I$, where I is 2×2 identity matrix. Define $\text{Tr}(A)$ = sum of diagonal elements of A and $|A|$ = determinant of matrix A .

Statement-1: $\text{Tr}(A) = 0$.

Statement-2: $|A| = 1$.

(1) Statement-1 is true, Statement-2 is true; Statement-2 is **not** a correct explanation for Statement-1.

(2) Statement-1 is true, Statement-2 is false.

(3) Statement-1 is false, Statement-2 is true.

(4) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

Sol.: Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $a, b, c, d \neq 0$

$$\Rightarrow \begin{bmatrix} a^2+bc & ab+bd \\ ac+cd & bc+d^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \Rightarrow ab+bd=0$$

$$\Rightarrow a+d=0 \Rightarrow \text{Tr}(A)=0$$

$$\text{Also } a^2+bc=bc+d^2=1$$

$$|A| = ad-bc = -a^2-bc = -1$$

Correct choice: (2)

76. Let $f : R \rightarrow R$ be a continuous function defined by $f(x) = \frac{1}{e^x + 2e^{-x}}$.

Statement-1: $f(c) = \frac{1}{3}$, for some $c \in R$.

Statement-2: $0 < f(x) \leq \frac{1}{2\sqrt{2}}$, for all $x \in R$.

(1) Statement-1 is true, Statement-2 is true; Statement-2 is **not** a correct explanation for Statement-1.

(2) Statement-1 is true, Statement-2 is false.

(3) Statement-1 is false, Statement-2 is true.

(4) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

Sol.: $\frac{e^x + 2e^{-x}}{2} \geq \sqrt{e^x \cdot 2e^{-x}}$

$$\Rightarrow 0 < \frac{1}{e^x + 2e^{-x}} \leq \frac{1}{2\sqrt{2}}$$

As $f(x)$ is continuous, so it will take every value between $\left(0, \frac{1}{2\sqrt{2}}\right]$ atleast once.

So for some c , $f(c) = \frac{1}{3}$

Correct choice: (4)

77. For a regular polygon, let r and R be the radii of the inscribed and the circumscribed circles. A **false** statement among the following is

(1) There is a regular polygon with $\frac{r}{R} = \frac{1}{\sqrt{2}}$

(2) There is a regular polygon with $\frac{r}{R} = \frac{2}{3}$

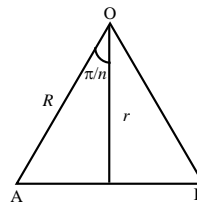
(3) There is a regular polygon with $\frac{r}{R} = \frac{\sqrt{3}}{2}$

(4) There is a regular polygon with $\frac{r}{R} = \frac{1}{2}$

Sol.: If O is centre of polygon and AB is one of the side, then by figure:

$$\cos \frac{\pi}{n} = \frac{r}{R}$$

$$\Rightarrow \frac{r}{R} = \frac{1}{2}, \frac{1}{\sqrt{2}}, \frac{\sqrt{3}}{2} \text{ for } n = 3, 4, 6 \text{ respectively}$$



Correct choice: (2)

*78. If α and β are the roots of the equation $x^2 - x + 1 = 0$, then $\alpha^{2009} + \beta^{2009} =$

(1) -1

(2) 1

(3) 2

(4) -2

Sol.: $\alpha = -\omega, \beta = -\omega^2$

$$\Rightarrow \alpha^{2009} + \beta^{2009} = (-\omega)^{2009} + (-\omega^2)^{2009} = -\omega^2 - \omega = 1$$

Correct choice: (2)

*79. The number of complex numbers z such that $|z-1| = |z+1| = |z-i|$ equals

(1) 1

(2) 2

(3) ∞

(4) 0

Sol.: Only one solution and that is circumcentre of triangle formed by $(1, 0)$, $(0, 1)$ and $(-1, 0)$.

Correct choice: (1)

*80. A line AB in three-dimensional space makes angles 45° and 120° with the positive x -axis and the positive y -axis respectively. If AB makes an acute angle θ with the positive z -axis, then θ equals

(1) 45°

(2) 60°

(3) 75°

(4) 30°

Sol.: $\cos^2 45^\circ + \cos^2 120^\circ + \cos^2 \theta = 1 \Rightarrow \frac{1}{2} + \frac{1}{4} + \cos^2 \theta = 1 \Rightarrow \cos^2 \theta = \frac{1}{4} \Rightarrow \cos \theta = \pm \frac{1}{2} \Rightarrow \theta = 60^\circ \text{ or } 120^\circ$

Correct choice: (2)

***81.** The line L given by $\frac{x}{5} + \frac{y}{b} = 1$ passes through the point $(13, 32)$. The line K is parallel to L and has the equation $\frac{x}{c} + \frac{y}{3} = 1$.

Then the distance between L and K is

(1) $\sqrt{17}$ (2) $\frac{17}{\sqrt{15}}$ (3) $\frac{23}{\sqrt{17}}$ (4) $\frac{23}{\sqrt{15}}$

Sol.: As L is passing through $(13, 32)$

$$\therefore \frac{13}{5} + \frac{32}{b} = 1 \Rightarrow b = -20$$

So equation of L is $4x - y - 20 = 0$ i.e., $\frac{x}{-3/4} + \frac{y}{3} + \frac{20}{3} = 0 \Rightarrow c = \frac{-3}{4}$

Distance between K and L is $= \frac{|20+3|}{\sqrt{16+1}} = \frac{23}{\sqrt{17}}$

Correct choice: (3)

***82.** A person is to count 4500 currency notes. Let a_n denote the number of notes he counts in the n^{th} minute. If $a_1 = a_2 = \dots = a_{10} = 150$ and a_{10}, a_{11}, \dots are in an AP with common difference -2 , then the time taken by him to count all notes is

(1) 34 minutes (2) 125 minutes (3) 135 minutes (4) 24 minutes

Sol.: $a_{11} = 148$ and $a_{11} + a_{12} + a_{13} + \dots + a_n = 3000$

$$\Rightarrow \frac{n-10}{2} [2(148) + (n-11)(-2)] = 3000 \Rightarrow n^2 - 169n + 4590 = 0 \Rightarrow (n-135)(n-34) = 0 \Rightarrow n = 34, 135$$

But n can not be 135, because a_{135} is negative.

Correct choice: (1)

83. Let $f: R \rightarrow R$ be a positive increasing function with $\lim_{x \rightarrow \infty} \frac{f(3x)}{f(x)} = 1$. Then $\lim_{x \rightarrow \infty} \frac{f(2x)}{f(x)} =$

(1) $\frac{2}{3}$ (2) $\frac{3}{2}$ (3) 3 (4) 1

Sol.: Clearly $\lim_{x \rightarrow \infty} f(x) = \lim_{x \rightarrow \infty} f(2x) = \lim_{x \rightarrow \infty} f(3x) \Rightarrow \lim_{x \rightarrow \infty} \frac{f(2x)}{f(x)} = 1$

Correct choice: (4)

84. Let $p(x)$ be a function defined on R such that $p'(x) = p'(1-x)$, for all $x \in [0, 1]$, $p(0) = 1$ and $p(1) = 41$. Then $\int_0^1 p(x) dx$ equals

(1) 21 (2) 41 (3) 42 (4) $\sqrt{41}$

Sol.: $p(x) + p(1-x) = c$ or $p(x) + p(1-x) = 42$

$$\text{Let } I = \int_0^1 p(x) dx = \int_0^1 p(1-x) dx = \int_0^1 (42 - p(x)) dx$$

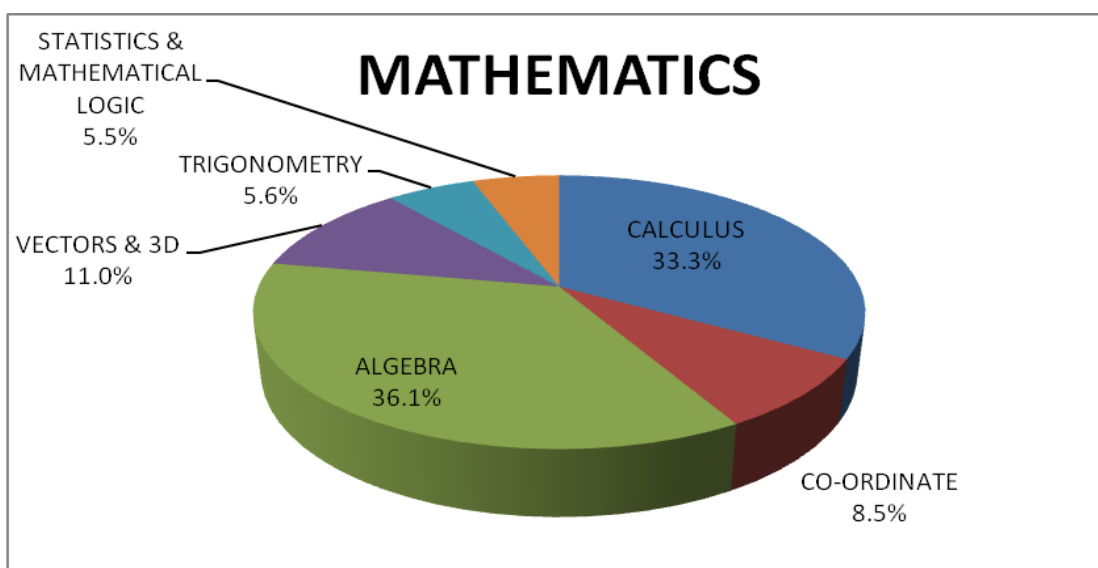
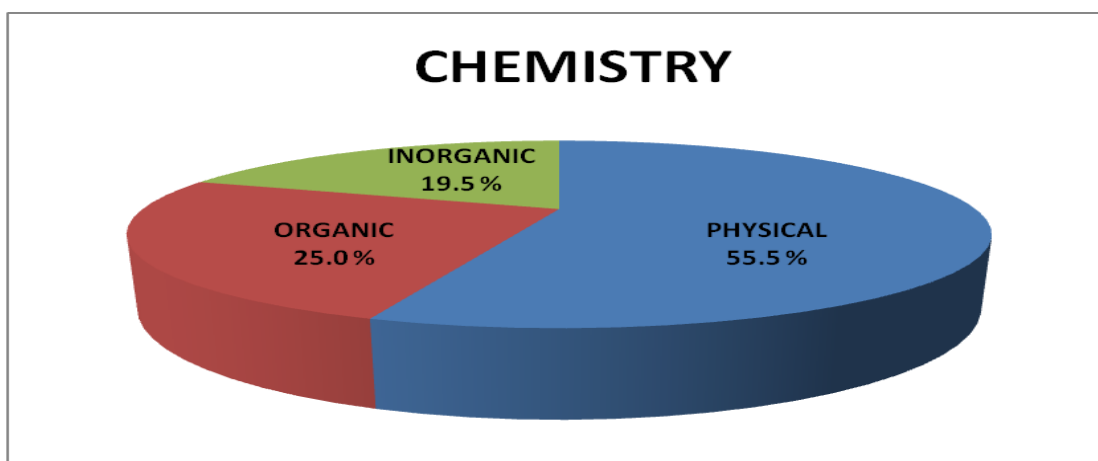
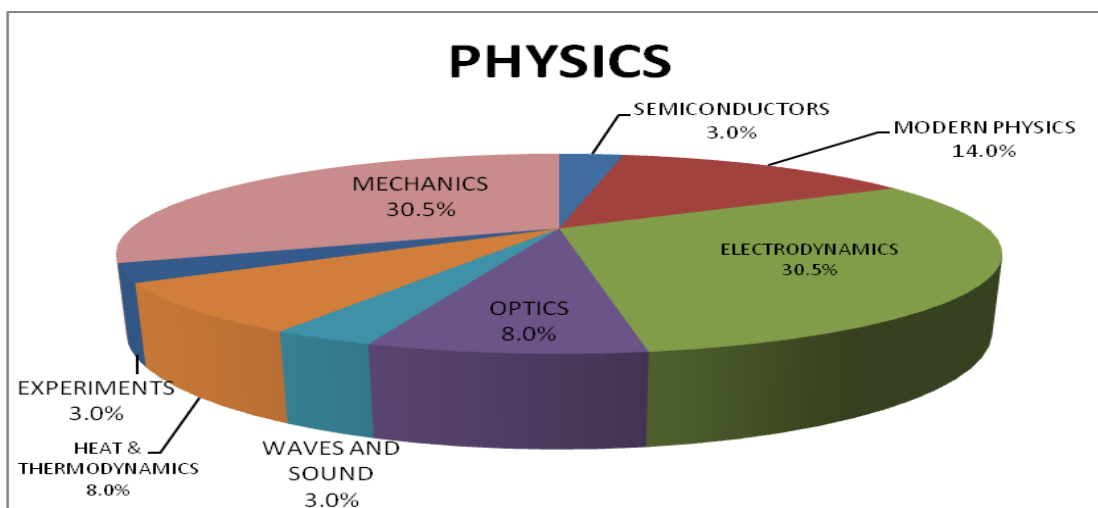
$$I = 42 - I \Rightarrow I = 21$$

Correct choice: (1)

85. Let $f: (-1, 1) \rightarrow R$ be a differentiable function with $f(0) = -1$ and $f'(0) = 1$. Let $g(x) = [f(2f(x)+2)]^2$. Then $g'(0) =$

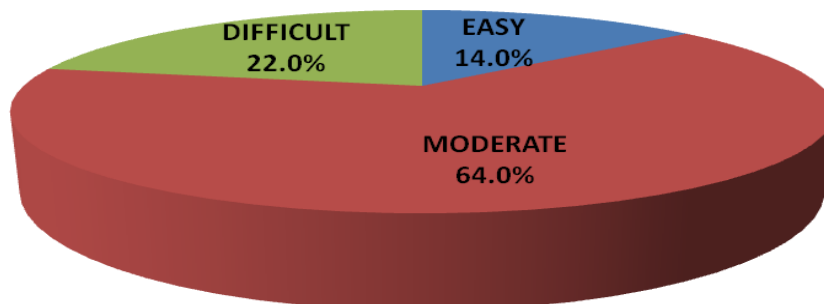
(1) -4 (2) 0 (3) -2 (4) 4

TOPICWISE

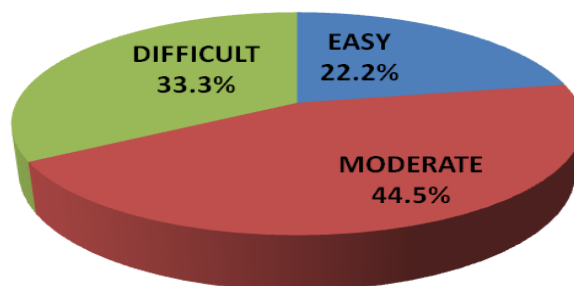


LEVEL OF DIFFICULTY

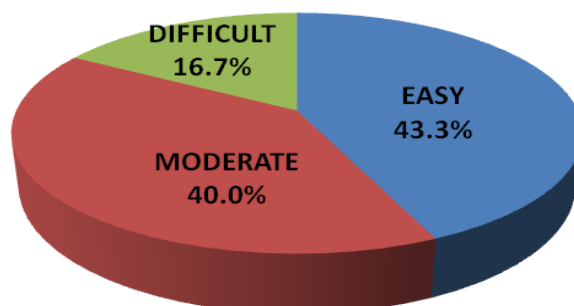
PHYSICS



CHEMISTRY

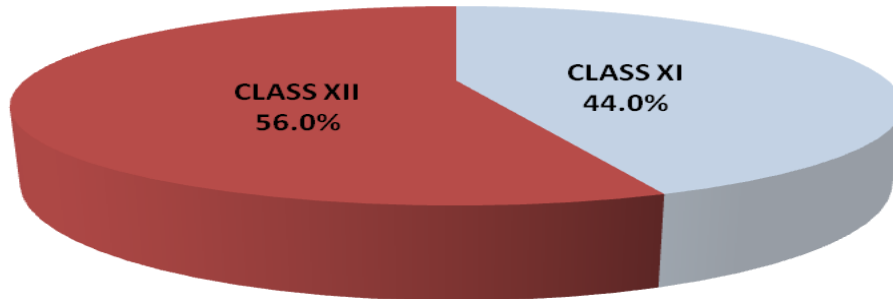


MATHEMATICS

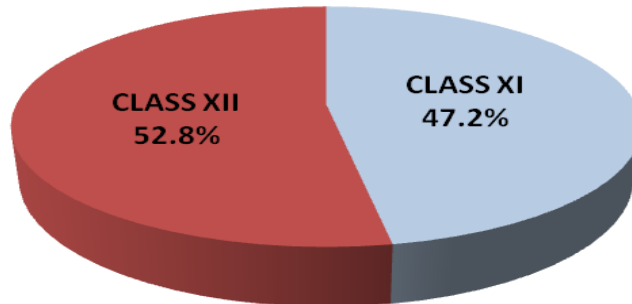


CLASSWISE

PHYSICS



CHEMISTRY



MATHEMATICS

