BRILLIANT® TUTORIALS

(Code: 6)

SOLUTIONS TO IIT-JEE 2011
Paper-I

Time: 3 Hours **Maximum Marks: 240**

Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose.

INSTRUCTIONS

A. General:

1. The question paper CODE is printed on the right hand top corner of this sheet.
2. No additional sheets will be provided for rough work.
3. Blank papers, clipboards, log tables, slide rules, calculators, cellular phones, pagers and electronic gadgets are NOT allowed.
4. Write your name and registration number in the space provided on the back page of this booklet.
5. The answer sheet, a machine-gradable Optical Response Sheet (ORS), is provided separately.
6. DO NOT TAMPER WITH/MULTILATE THE ORS OR THE BOOKLET.
7. Do not break the seals of the question-paper booklet before being instructed to do so by the invigilators.
8. This Question Paper contains 69 questions.
9. On breaking the seals, please check that all the questions are legible.

B. Filling the Right Part of the ORS:

10. The ORS also has a CODE printed on its Left and Right parts.
11. Make sure the CODE on the ORS is the same as that on this booklet. **If the codes do not match, ask for a change of the booklet.**
12. Write your Name, Registration No. and the name of centre and sign **with pen** in the boxes provided. **Do not write them anywhere else.** Darken the appropriate bubble **UNDER** each digit of your registration No. with a **good quality HB pencil.**

C. Question paper format and Marking Scheme:

13. The question paper consists of **3 parts** (Chemistry, Physics and Mathematics). Each part consists of **four sections**.
14. In **Section I** (Total Marks: 21), for each question you will be awarded **3 marks** if you darken **ONLY** the bubble corresponding to the correct answer and **zero marks** if no bubble is darkened. In all other cases, **minus one (–1) mark** will be awarded.
15. In **Section II** (Total Marks: 16), for each question you will be awarded **4 marks** if you darken **ALL** the bubble(s) corresponding to the correct answer(s) **ONLY** and **zero marks** otherwise. There are **no negative marks** in this section.
16. In **Section III** (Total Marks : 15), for each question you will be awarded **3 marks** if you darken **ONLY** the bubble corresponding to the correct answer and **zero marks** if no bubble is darkened. In all other cases, **minus one (–1) mark** will be awarded.
17. In **Section IV** (Total Marks: 28), for each question you will be awarded **4 marks** if you darken **ONLY** the bubble corresponding to the correct answer and **zero marks** otherwise. There are **no negative marks** in this section.

SOLUTIONS TO IIT-JEE 2011
CHEMISTRY: Paper-I (Code: 6)

PART – I

Useful Data

$R = 8.314\ JK^{-1}\ mol^{-1}$ or $8.206 \times 10^{-2}\ L\ atm\ K^{-1}\ mol^{-1}$	$1\ F = 96500\ C\ mol^{-1}$
$h = 6.626 \times 10^{-34}\ Js$	$1\ eV = 1.602 \times 10^{-19}\ J$
$c = 3.0 \times 10^{8}\ m\ s^{-1}$	$N_A = 6.022 \times 10^{23}$

SECTION – I (Total Marks: 21)
(Single Correct Answer Type)

This section contains **7 multiple choice questions**. Each question has four choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.

Note: Questions with (*) mark are from syllabus of class XI.

***1.** Among the following compounds, the most acidic is

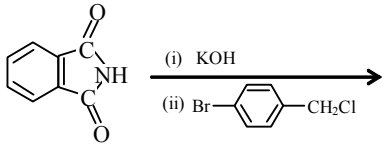
(A) p-nitrophenol (B) p-hydroxybenzoic acid

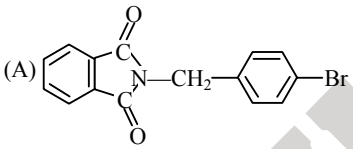
(C) o-hydroxybenzoic acid (D) p-toluic acid

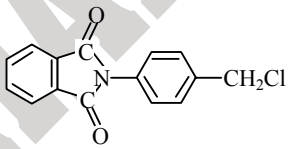
Sol.: o-Hydroxybenzoic acid is most acidic among the given compounds due to ortho effect and stabilization of its conjugate base by intramolecular hydrogen bonding.

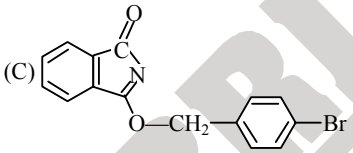
Correct choice: (C)

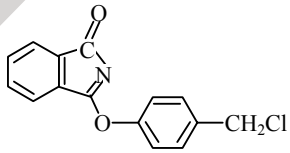
2. The major product of the following reaction is

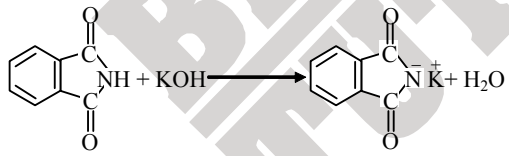


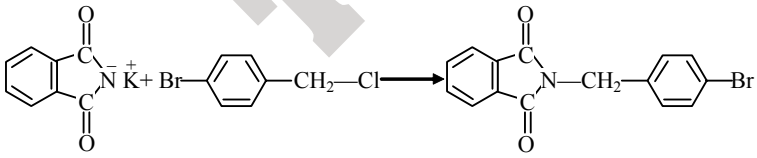
(A) 

(B) 

(C) 

(D) 

Sol.: 



Correct choice: (A)

3. Extra pure N_2 can be obtained by heating

(A) NH_3 with CuO (B) NH_4NO_3

(C) $(NH_4)_2Cr_2O_7$ (D) $Ba(N_3)_2$

Sol.: Nitrogen gas is obtained in the following reactions:

$$2NH_3(g) + 3CuO(s) \xrightarrow{\Delta} N_2(g) + 3Cu(s) + 3H_2O(g)$$

$$(NH_4)_2Cr_2O_7(s) \xrightarrow{\Delta} N_2(g) + Cr_2O_3(s) + 4H_2O(g)$$

$$Ba(N_3)_2(s) \xrightarrow{\Delta} Ba(s) + 3N_2(g)$$

Extra pure N_2 is obtained by heating $Ba(N_3)_2$ in vacuum as no gaseous by-product is formed in the reaction.

Correct choice: (D)

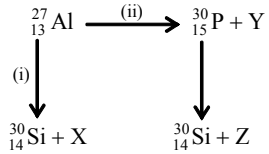
4. Geometrical shapes of the complexes formed by the reaction of Ni^{2+} with Cl^-, CN^- and H_2O, respectively, are

(A) octahedral, tetrahedral and square planar.
(B) tetrahedral, square planar and octahedral.
(C) square planar, tetrahedral and octahedral.
(D) octahedral, square planar and octahedral.

Sol.: Ni^{2+} forms $[NiCl_4]^{2-}$ with Cl^-, whereas it forms $[Ni(CN)_4]^{2-}$ with CN^- and $[Ni(H_2O)_6]^{2+}$ with H_2O. The hybridisation of Ni^{2+} in $[NiCl_4]^{2-}$, $Ni(CN)_4]^{2-}$ and $[Ni(H_2O)_6]^{2+}$ is sp^3, dsp^2 and sp^3d^2 respectively. Hence, the shapes of $[NiCl_4]^{2-}$, $[Ni(CN)_4]^{2-}$ and $[Ni(H_2O)_6]^{2+}$ are tetrahedral, square planar and octahedral respectively.

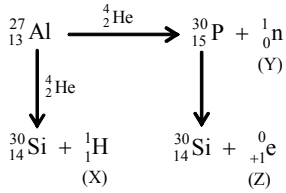
Correct choice: (B)

5. Bombardment of aluminium by α-particle leads to its artificial disintegration in two ways, (i) and (ii) as shown. Products X, Y and Z respectively are



(A) proton, neutron, positron
(B) neutron, positron, proton
(C) proton, positron, neutron
(D) positron, proton, neutron

Sol.:



Correct choice: (A)

6. Dissolving 120 g of urea (molecular weight 60) in 1000 g of water gave a solution of density 1.15 g/mL. The molarity of the solution is

(A) 1.78 M
(B) 2.00 M
(C) 2.05 M
(D) 2.22 M

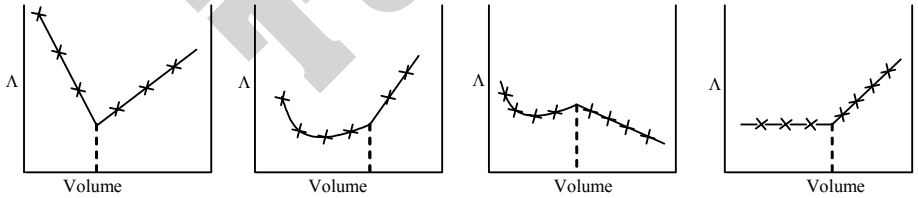
Sol.: Mass of solution = 1120 g.

Volume of solution = $\frac{1120}{1.15}$ ml . ; Number of moles of solute = $\frac{120}{60} = 2$.

Molarity of solution = $\frac{2 \times 1000 \times 1.15}{1120} = 2.05$ M

Correct choice: (C)

7. $AgNO_3$ (aq.) was added to an aqueous KCl solution gradually and the conductivity of the solution was measured. The plot of conductance (Λ) versus the volume of $AgNO_3$ is



(A) (P)
(B) (Q)
(C) (R)
(D) (S)

Sol.: When $AgNO_3$ is added to KCl, the first part of the curve shows no increase in conductivity as there is only replacement of Cl^- ions with NO_3^- ions. (Ionic conductivity of Cl^- ions is 76 and that of NO_3^- ions is 71). As the AgCl is precipitated, it does not contribute to conductivity. The second part of the curve i.e., after the point of equivalence, conductivity increases because of increase in concentration of Ag^+ and NO_3^- ions.

Correct choice: (D)

SECTION – II (Total Marks: 16)
(Multiple Correct Answers Type)

This section contains **4 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONE OR MORE** may be correct.

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8. The correct statement(s) pertaining to the adsorption of a gas on a solid surface is (are)

(A) Adsorption is always exothermic.

(B) Physisorption may transform into chemisorption at high temperature.

(C) Physisorption increases with increasing temperature but chemisorption decreases with increasing temperature.

(D) Chemisorption is more exothermic than physisorption, however it is very slow due to higher energy of activation.

Sol.: Adsorption of a gas on the surface of a solid is generally exothermic but not always. e.g. adsorption of H_2 on glass surface is endothermic. Physiosorption may transform into chemisorption at high temperature because chemisorption involves energy barrier. Chemisorption is more exothermic than physiosorption as the formation of covalent bonds in former case results in release of energy.

Correct choice: (B) & (D)

9. Extraction of metal from the ore **cassiterite** involves

(A) carbon reduction of an oxide ore (B) self-reduction of a sulphide ore

(C) removal of copper impurity (D) removal of iron impurity

Sol.: SnO_2 is reduced to the metal by heating with carbon at 1200–1300°C in an electric furnace. The crude tin contains impurities. It is purified by liquation and poling. The crude metal is heated gently on a sloping hearth when pure tin flows down leaving behind iron and copper as impurities.

Correct choice: (A), (C) & (D)

***10.** According to kinetic theory of gases

(A) collisions are always elastic.

(B) heavier molecules transfer more momentum to the wall of the container.

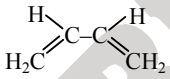
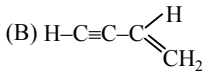
(C) only a small number of molecules have very high velocity.

(D) between collisions, the molecules move in straight lines with constant velocities.

Sol.: As per kinetic theory of gases, gas molecules are presumed to be point particles, moving in random motion colliding with each other as well as with the walls of container. Collisions between such particles are taken to be perfectly elastic. Molecules travel in straight lines in between the collisions. According to Maxwell's distribution only a small number of molecules have very high velocity. Heavier molecules transfer more momentum to the wall of container.

Correct choice: (A), (B), (C) & (D)

***11.** Amongst the given options, the compound(s) in which all the atoms are in one plane in all the possible conformations (if any), is (are)

(A)  (B) 

(C) $H_2C{=}C{=}O$ (D) $H_2C{=}C{=}CH_2$

Sol.: Both s-cis and s-trans structures of 1,3-butadiene are planar. Structures (B) and (C) are also planar. Propadiene has terminal hydrogens lying perpendicular to each other and hence don't lie in the same plane.

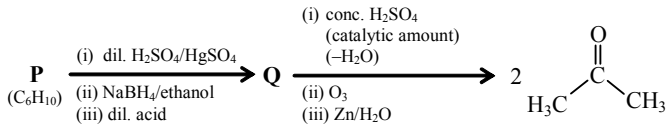
Correct choice: (A), (B) & (C)

SECTION – III (Total Marks: 15)
(Paragraph Type)

This section contains **2 paragraphs**. Based upon one of the paragraphs **2 multiple choice questions** and based on the other paragraph **3 multiple choice questions** have to be answered. Each of these questions has four choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

Paragraph for Question Nos. 12 to 13

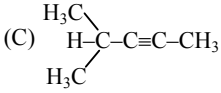
An acyclic hydrocarbon **P**, having molecular formula C_6H_{10}, gave acetone as the only organic product through the following sequence of reaction, in which **Q** is an intermediate organic compound.

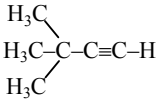


***12.** The structure of compound **P** is

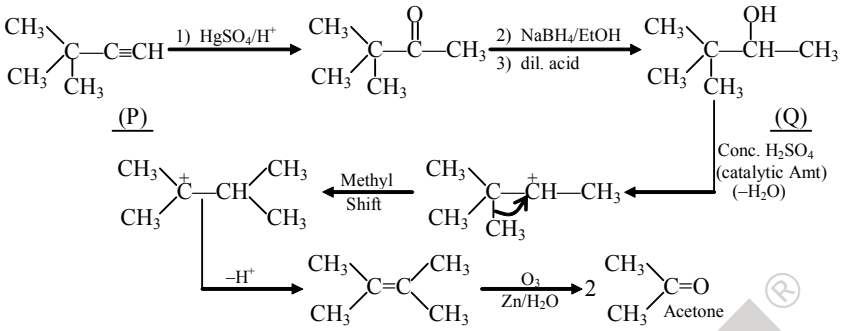
(A) $CH_3CH_2CH_2CH_2–C\equiv C–H$

(B) $H_3CH_2C–C\equiv C–CH_2CH_3$

(C) 

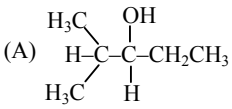
(D) 

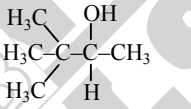
Sol.: **Solution for Question No. 12 & 13**

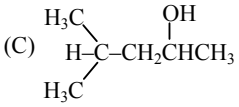


Correct choice: (D)

***13.** The structure of the compound **Q** is

(A) 

(B) 

(C) 

(D) 

Correct choice: (B)

Paragraph for Question Nos. 14 to 16

When a metal rod **M** is dipped into an aqueous colourless concentrated solution of compound **N**, the solution turns light blue. Addition of aqueous NaCl to the blue solution gives white precipitate **O**. Addition of aqueous NH_3 dissolves **O** and gives an intense blue solution.

14. The metal rod **M** is

(A) Fe

(B) Cu

(C) Ni

(D) Co

Sol.:

(1) $\underset{(M)}{Cu} + 2\underset{(N)}{AgNO_3} \longrightarrow \underset{\text{Blue Solution}}{Cu(NO_3)_2} + 2Ag$

As $AgNO_3$ solution is concentrated so, on addition of NaCl.

$AgNO_3 + NaCl \longrightarrow \underset{\substack{\text{White ppt}\\(O)}}{AgCl} + NaNO_3$

(2) On addition of aqueous NH_3,

$Cu^{+2} + 4NH_3 \longrightarrow \underset{\text{Intense blue solution}}{[Cu(NH_3)_4]^{+2}}$

$Ag^{+} + 2NH_3 \longrightarrow \underset{\text{Soluble solution}}{[Ag(NH_3)_2]^{+}}$

Correct choice: (B)

15. The compound **N** is

(A) $AgNO_3$ (B) $Zn(NO_3)_2$

(C) $Al(NO_3)_3$ (D) $Pb(NO_3)_2$

Sol.: **Correct choice: (A)**

16. The final solution contains

(A) $[Pb(NH_3)_4]^{2+}$ and $[CoCl_4]^{2-}$ (B) $[Al(NH_3)_4]^{3+}$ and $[Cu(NH_3)_4]^{2+}$

(C) $[Ag(NH_3)_2]^{+}$ and $[Cu(NH_3)_4]^{2+}$ (D) $[Ag(NH_3)_2]^{+}$ and $[Ni(NH_3)_6]^{2+}$

Sol.: **Correct choice: (C)**

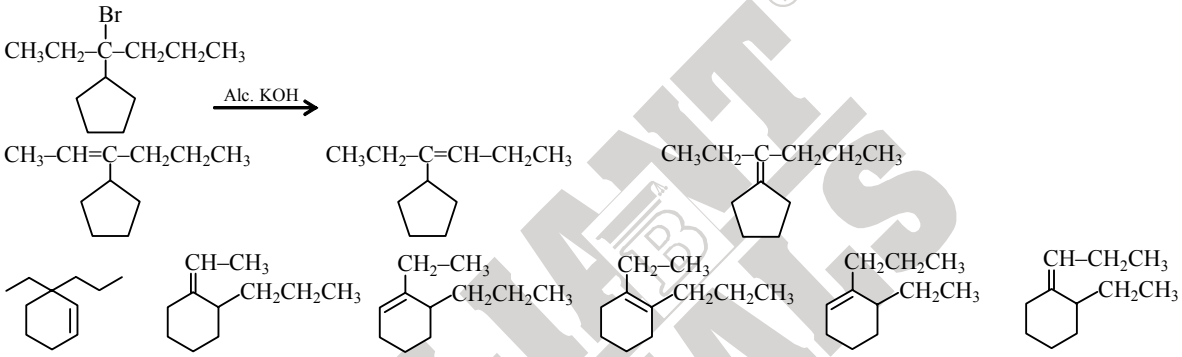
SECTION–IV (Total Marks: 28)

Integer Answer Type

This section contains **7 questions**. The answer to each questions is a **single digit integer** ranging from 0 to 9. The bubble corresponding to the correct answer is to be darkened in the ORS.

***17.** The total number of alkenes possible by dehydrobromination of 3-bromo-3-cyclopentylhexane using alcoholic KOH is

Sol.: The following possible alkenes will be formed for the given reaction:



∴ The answer is **9**.

18. A decapeptide (molecular weight 796) on complete hydrolysis gives glylcine (molecular weight 75), alanine and phenylalanine. Glycine contributes 47.0% to the total weight of the hydrolysed products. The number of glycine units present in the decapeptide is

Sol.: For hydrolysis of a peptide number of water molecules required = (n –1).

Where n is number of peptide linkages.

So, for a decapeptide total H_2O molecules needed = 9.

Now total molecular mass of all amino acids forming the decapeptide = 796 + 9 (18)

= 796 + 162

= 958

Mass contribution of Glycine = 47 % of 958

= 450.26

Number of glycine units in the decapeptide = $\frac{450.26}{75} = 6$

∴ The answer is **6.**

***19.** To an evacuated vessel with movable piston under external pressure of 1 atm., 0.1 mol of He and 1.0 mol of an unknown compound (vapour pressure 0.68 atm. at 0°C) are introduced. Considering the ideal gas behaviour, the total volume (in litre) of the gases at 0°C is close to

Sol.: External pressure = 1 atm.

Vapour pressure of unknown compound at 0°C = 0.68 atm.

Partial pressure of He = 1 – 0.68 = 0.32 atm.

If V is the total volume of gases at 0°C then

$0.32 \times V = 0.1 \times 0.0821 \times 273$

∴ V = 7 litres.

∴ The answer is **7**.

***20.** The work function (ϕ) of some metals is listed below. The number of metals which will show photoelectric effect when light of 300 nm wavelength falls on the metal is

Metal	Li	Na	K	Mg	Cu	Ag	Fe	Pt	W
ϕ (eV)	2.4	2.3	2.2	3.7	4.8	4.3	4.7	6.3	4.75

Sol.: Energy of incident radiation $= \frac{hc}{\lambda} = \frac{6.634\times10^{-34}\times3\times10^{8}}{300\times10^{-9}}$

$= 6.6 \times 10^{-19}$ J

$= \frac{6.6\times10^{-19}}{1.6\times10^{-19}} = 4.1$ eV

Now, Li, Na, K, Mg have work function (ϕ) less than 4.1 eV. So, they will show photoelectric emission.

$\therefore$ The answer is **4**.

***21.** The maximum number of electrons that can have principal quantum number, n = 3, and spin quantum number, $m_s = -1/2$, is

Sol.: Total number of electrons in 3rd shell $= 2 \times (3^2) = 18$.

Now, 9 electrons have $m_s = +1/2$ and 9 electrons have $m_s = -1/2$.

$\therefore$ The answer is **9**.

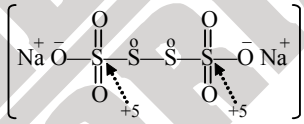
22. Reaction of Br_2 with Na_2CO_3 in aqueous solution gives sodium bromide and sodium bromate with evolution of CO_2 gas. The number of sodium bromide molecules involved in the balanced chemical equation is

Sol.: $3Na_2CO_3 + 3Br_2 \longrightarrow 5NaBr + NaBrO_3 + 3CO_2$

This is disproportionation reaction of bromine. Five molecules of NaBr are involved in the reaction.

$\therefore$ The answer is **5**.

23. The difference in the oxidation numbers of the two types of sulphur atoms in $Na_2S_4O_6$ is

Sol.: $Na_2S_4O_6$ (Sodium tetrathionate) has structure 

Difference between oxidation numbers of the two different sulphur atoms = 5 – 0 = 5

$\therefore$ The answer is **5**.

SOLUTIONS TO IIT-JEE 2011
PHYSICS: Paper-I (Code: 6)

PART – II

SECTION – I (Total Marks: 21)
(Single Correct Answer Type)

This Section contains **7 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE is correct**.

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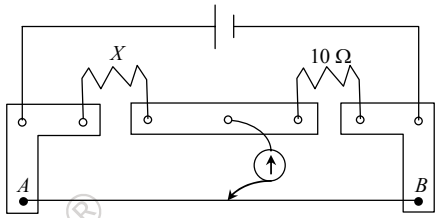
24. A meter bridge is set-up as shown, to determine an unknown resistance '*X*' using a standard 10 ohm resistor. The galvanometer shows null point when tapping-key is at 52 cm mark. The end-corrections are 1 cm and 2 cm respectively for the ends *A* and *B*. The determined value of '*X*' is



(A) 10.2 ohm (B) 10.6 ohm

(C) 10.8 ohm (D) 11.1 ohm

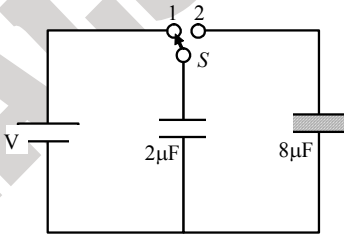
Sol.: Effective length of the wire $l_0 = 100 + (1 + 2) = 103$ cm

Balancing length (l) = 52 + 1 = 53 cm

$$X = \frac{Rl}{l_0 - l} = \frac{10 \times 53}{103 - 53} = \frac{530}{50} = 10.6\ \Omega$$

Correct choice: (B)

25. A 2 μF capacitor is charged as shown in figure. The percentage of its stored energy dissipated after the switch *S* is turned to position 2 is



(A) 0 % (B) 20 %

(C) 75 % (D) 80 %

Sol.: Initial charge $q_{2\mu f} = 2V$

After shifting,

common potential = $\frac{V}{5}$

$$U_i = \frac{1}{2} \times 2 \times V^2 = V^2$$

$$U_f = \frac{1}{2} \times 10 \times \left(\frac{V}{5}\right)^2 = \frac{V^2}{5}$$

$\therefore$ % loss of energy $\frac{U_i - U_f}{U_i} \times 100 = 80\%$

Correct choice: (D)

***26.** A police car with a siren of frequency 8 kHz is moving with uniform velocity 36 km/hr towards a tall building which reflects the sound waves. The speed of sound in air is 320 m/s. The frequency of the siren heard by the car driver is

(A) 8.50 kHz (B) 8.25 kHz (C) 7.75 kHz (D) 7.50 kHz

Sol.: $\nu' = 8\ \text{kHz}\left[\frac{320 + 10}{320 - 10}\right] = 8.5\ \text{kHz}$

Correct choice (A)

***27.** 5.6 liter of helium gas at STP is adiabatically compressed to 0.7 liter. Taking the initial temperature to be T_1, the work done in the process is

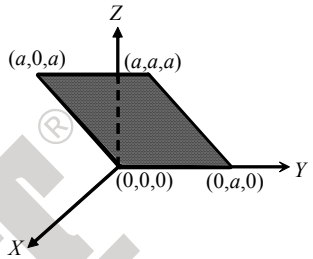
(A) $\frac{9}{8}RT_1$ (B) $\frac{3}{2}RT_1$ (C) $\frac{15}{8}RT_1$ (D) $\frac{9}{2}RT_1$

Sol.: $T_2 = T_1\left[\frac{V_1}{V_2}\right]^{\gamma-1} = T_1\left[\frac{5.6}{0.7}\right]^{\frac{5}{3}-1} = 4\,T_1$

$W = |-nC_v\,\Delta T| = \left(\frac{5.6}{22.4}\right)\times\frac{3}{2}R\times(4T_1 - T_1) = \frac{9}{8}RT_1$

Correct choice: (A)

28. Consider an electric field $\vec{E} = E_0\hat{x}$, where E_0 is a constant. The flux through the shaded area (as shown in the figure) due to this field is



(A) $2E_0\,a^2$ (B) $\sqrt{2}E_0\,a^2$

(C) $E_0\,a^2$ (D) $\frac{E_0\,a^2}{\sqrt{2}}$

Sol.: Projected area on the $y-z$ plane $= a^2$

$\therefore$ flux $= E_0\,a^2$

Correct choice: (C)

29. The wavelength of the first spectral line in the Balmer series of hydrogen atom is 6561 Å. The wavelength of the second spectral line in the Balmer series of singly-ionized helium atom is

(A) 1215 Å (B) 1640 Å (C) 2430 Å (D) 4687 Å

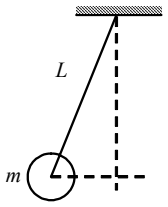
Sol.: $\frac{1}{\lambda_1} = R(1)^2\left[\frac{1}{4}-\frac{1}{9}\right] = \frac{5}{36}R$

$\frac{1}{\lambda_2} = R(2)^2\left[\frac{1}{4}-\frac{1}{16}\right] = \frac{3}{4}R$

$\frac{\lambda_2}{\lambda_1} = \frac{5}{27} \Rightarrow \lambda_2 = 1215$ Å

Correct choice: (A)

***30.** A ball of mass (m) 0.5 kg is attached to the end of a string having length (L) 0.5 m. The ball is rotated on a horizontal circular path about vertical axis. The maximum tension that the string can is bear is 324 N. The maximum possible value of angular velocity of ball (in radian/s) is



(A) 9 (B) 18 (C) 27 (D) 36

Sol.: $T\cos\theta = mg$

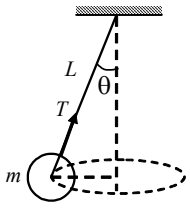
$T\sin\theta = ml\,\sin\theta\,\omega^2 \;\therefore\; T = ml\omega^2$

$T_{max} = ml\omega_{max}^2$

$324 = 0.5\times 0.5\;\omega_{max}^2$

$\omega_{max}^2 = \frac{324}{25}\times 100$

$\omega_{max} = 18\times 2 = 36$



Correct choice: (D)

SECTION – II (Total Marks: 16)
(Multiple Correct Answer Type)

This section contains **4 multiple choice questions.** Each question has four choices (A), (B), (C) and (D) out of which **ONE OR MORE** may be correct.

---

31. An electron and a proton are moving on straight parallel paths with same velocity. They enter a semi-infinite region of uniform magnetic field perpendicular to the velocity. Which of the following statement(s) is/are true?

(A) They will never come out of the magnetic field region.

(B) They will come out traveling along parallel paths.

(C) They will come out at the same time.

(D) They will come out at different times.

Sol.: Both follow semicircular path in the magnetic field for the time $t = \frac{\pi m}{qB}$

Correct choices: (B), (D)

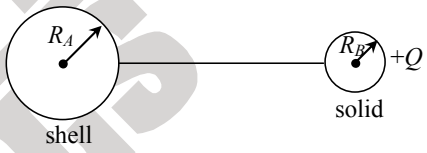
32. A spherical metal shell A of radius R_A and a solid metal sphere B of radius R_B ($< R_A$) are kept far apart and each is given charge '$+Q$'. Now they are connected by a thin metal wire. Then

(A) $E_A^{inside} = 0$

(B) $Q_A > Q_B$

(C) $\frac{\sigma_A}{\sigma_B} = \frac{R_B}{R_A}$

(D) $E_A^{on\ surface} < E_B^{on\ surface}$



Sol.: (A) Electric field inside the spherical shell is zero

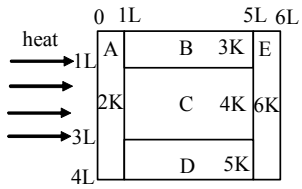
(B) $\frac{Q_A}{Q_B} = \frac{C_A V}{C_B V} = \frac{R_A}{R_B}$

(C) $V_A = V_B \Rightarrow \sigma_A R_A = \sigma_B R_B \Rightarrow \frac{\sigma_A}{\sigma_B} = \frac{R_B}{R_A}$

(D) $\frac{E_A}{E_B} = \frac{\sigma_A}{\sigma_B} = \frac{R_B}{R_A}$

Correct choices: (A), (B), (C), (D)

***33.** A composite block is made of slabs A, B, C, D and E of different thermal conductivities (given in terms of a constant K) and sizes (given in terms of length, L) as shown in the figure. All slabs are of same width. Heat 'Q' flows only from left to right through the blocks. Then in steady state



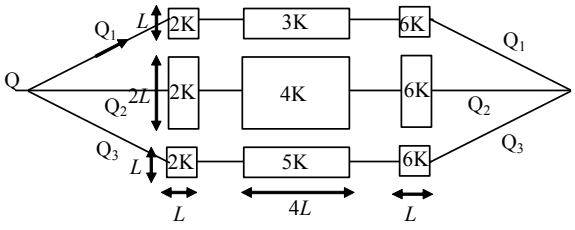
(A) heat flow through A and E slabs are same.

(B) heat flow through slab E is maximum.

(C) temperature difference across slab E is smallest.

(D) heat flow through C = heat flow through B + heat flow through D.

Sol.: Equivalent diagram is shown.



Heat current $Q_1 = \Delta T\left[\frac{2KLd}{L} + \frac{3KLd}{4L} + \frac{6KLd}{L}\right] = \frac{35}{4}Kd\Delta T$

$Q_2 = \Delta T\left[\frac{2K(2L)d}{L} + \frac{4K(2L)d}{4L} + \frac{6K(2L)d}{L}\right] = 18Kd\Delta T$

$Q_3 = \Delta T\left[\frac{2KLd}{L} + \frac{5KLd}{4L} + \frac{6KLd}{L}\right] = \frac{37}{4}Kd\Delta T$

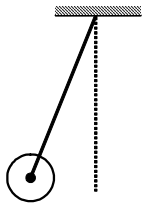
Clearly $Q_2 = Q_1 + Q_3$

(A) $Q_A = Q_E = Q_1 + Q_2 + Q_3$

(C) $\Delta T = QR = \frac{Ql}{KA}$ is least for E

Correct choices: (A), (C), (D) (Option (B) can be correct)

***34.** A metal rod of length 'L' and mass 'm' is pivoted at one end. A thin disk of mass 'M' and radius 'R' ($< L$) is attached at its center to the free end of the rod. Consider two ways the disk is attached: (case A) The disk is not free to rotate about its centre and (case B) the disk is free to rotate about its centre. The rod-disk system performs SHM in vertical plane after being released from the same displaced position. Which of the following statement(s) is (are) true?



(A) Restoring torque in case A = Restoring torque in case B.

(B) Restoring torque in case A < restoring torque in case B.

(C) Angular frequency for case A > Angular frequency for case B.

(D) Angular frequency for case A < Angular frequency for case B.

Sol.: Frequency of oscillation of physical pendulum is

$$T = 2\pi\sqrt{\frac{I}{mgd}}$$

$\because \quad I_A > I_B$

$\therefore \quad \omega_A < \omega_B$

Correct choices: (A), (D)

SECTION – III (Total Marks: 15)
(Paragraph Type)

This section contains **2 paragraphs**. Based upon one of the paragraphs **2 multiple choice questions** and based on the other paragraph **3 multiple choice questions** have to be answered. Each of these questions has four choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

Paragraph for Question Nos. 35 to 36

A dense collection of equal number of electrons and positive ions is called neutral plasma. Certain solids containing fixed positive ions surrounded by free electrons can be treated as neutral plasma. Let 'N' be the number density of free electrons, each of mass 'm'. When the electrons are subjected to an electric field, they are displaced relatively away from the heavy positive ions. If the electric field becomes zero, the electrons begin to oscillate about the positive ions with a natural angular frequency 'ω_p', which is called the plasma frequency. To sustain the oscillations, a time varying electric field needs to be applied that has an angular frequency ω, where a part of the energy is absorbed and a part of it is reflected. As ω approaches ω_p, all the free electrons are set to resonance together and all the energy is reflected. This is the explanation of high reflectivity of metals.

35. Taking the electronic charge as 'e' and the permittivity as 'ε_0', use dimensional analysis to determine the correct expression for ω_p.

(A) $\sqrt{\frac{Ne}{m\varepsilon_0}}$ (B) $\sqrt{\frac{m\varepsilon_0}{Ne}}$ (C) $\sqrt{\frac{Ne^2}{m\varepsilon_0}}$ (D) $\sqrt{\frac{m\varepsilon_0}{Ne^2}}$

Sol.: $\left[\sqrt{\frac{Ne^2}{m\varepsilon_0}}\right] = \left[\sqrt{\frac{L^{-3}A^2T^2}{M\ M^{-1}L^{-3}A^2T^4}}\right] = [T^{-1}]$

Correct choice: (C)

36. Estimate the wavelength at which plasma reflection will occur for a metal having the density of electrons $N \approx 4 \times 10^{27}$ m^{-3}. Take $\varepsilon_0 \approx 10^{-11}$ and $m \approx 10^{-30}$, where these quantities are in proper SI units.

(A) 800 nm (B) 600 nm (C) 300 nm (D) 200 nm

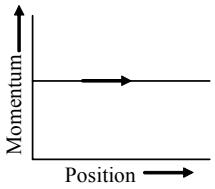
Sol.: $\sqrt{\frac{Ne^2}{m\varepsilon_0}} = \omega_p = 2\pi\nu = 2\pi\frac{c}{\lambda}$

$\lambda = 2\pi c\sqrt{\frac{m\varepsilon_0}{Ne^2}} \approx 600$ nm

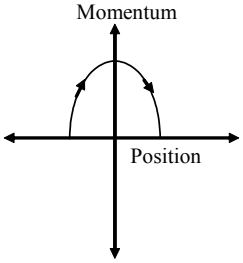
Correct choice: (B)

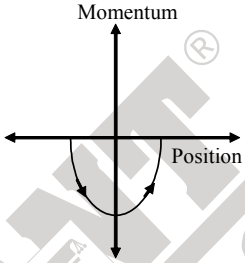
Paragraph for Question Nos. 37 to 39

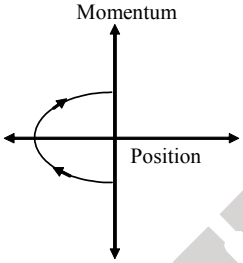
Phase space diagrams are useful tools in analyzing all kinds of dynamical problems. They are especially useful in studying the changes in motion as initial position and momentum are changed. Here we consider some simple dynamical systems in one-dimension. For such systems, phase space is a plane in which position is plotted along horizontal axis and momentum is plotted along vertical axis. The phase space diagram is $x(t)$ vs. $p(t)$ curve in this plane. The arrow on the curve indicates the time flow. For example, the phase space diagram for a particle moving with constant velocity is a straight line as shown in figure. We use the sign convention in which position or momentum upwards (or to right) is positive and downwards (or to left) is negative.

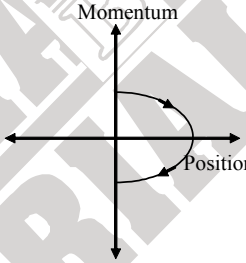


***37.** The phase space diagram for a ball thrown vertically up from ground is

(A)


(B)


(C)


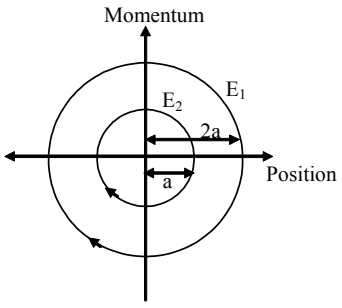
(D)


Sol.: Ball thrown vertically up.

$\Rightarrow \quad p = m\sqrt{V_0^2 - 2gy} \Rightarrow \quad p^2 = m^2\left(V_0^2 - 2gy\right)$

Correct choice: (D)

***38.** The phase space diagram for simple harmonic motion is a circle centered at the origin. In the figure, the two circles represent the same oscillator but for different initial conditions, and E_1 and E_2 are the total mechanical energies respectively. Then



(A) $E_1 = \sqrt{2}E_2$ (B) $E_1 = 2E_2$

(C) $E_1 = 4E_2$ (D) $E_1 = 16E_2$

Sol.: $E = \dfrac{p^2}{2m}$

$p^2 + x^2 = a^2$

$2mE + x^2 = a^2$

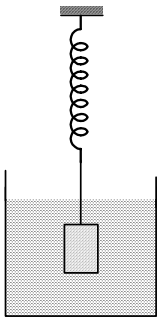
$2mE_2 = a^2$

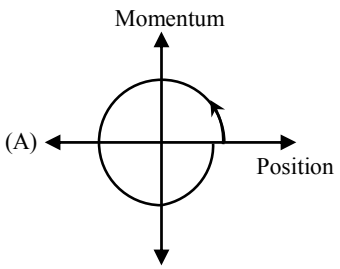
$2mE_1 = 4a^2$

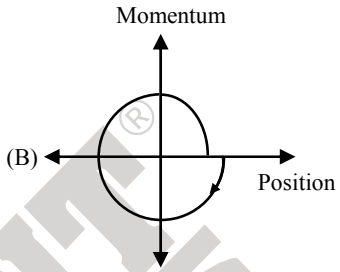
$\dfrac{E_1}{E_2} = 4$

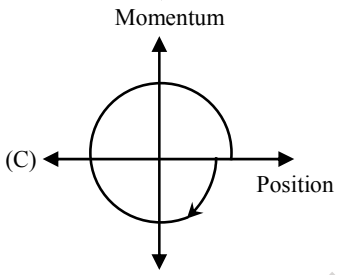
Correct choice: (C)

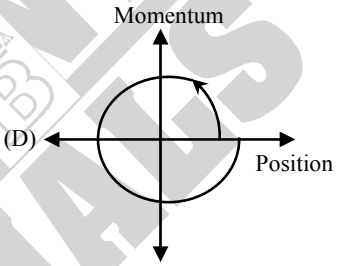
***39.** Consider the spring-mass system, with the mass submerged in water, as shown in the figure. The phase space diagram for one cycle of this system is











Sol.: From extreme positive position (of x), the momentum starts increasing in negative direction (with energy dissipation) and gradually returns back to less positive position as $p_f < p_i$ (due to energy dissipation).
Correct choice: (B)

SECTION – IV (Total Marks: 28)
(Integer Answer Type)

This section contains **7 questions**. The answer to each question is a single-digit integer, ranging from 0 to 9. The bubble corresponding to the correct answer is to be darkened in the ORS.

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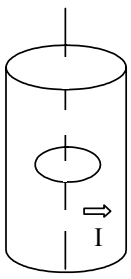
40. A long circular tube of length 10 m and radius 0.3 m carries a current I along its curved surface as shown. A wire-loop of resistance 0.005 ohm and of radius 0.1 m is placed inside the tube with its axis coinciding with the axis of the tube. The current varies as $I = I_0 \cos(300\ t)$ where I_0 is constant. If the magnetic moment of the loop is $N\ \mu_0\ I_0 \sin(300t)$; then 'N' is



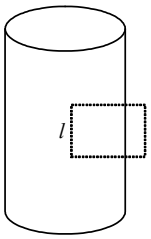
Sol.: From Ampere's circuital law,

$$\oint B.dl = \mu_0 \frac{I}{L} l$$

$$B = \frac{\mu_0 I}{L}$$

Now, $\phi = BA$

$$\phi = \frac{\mu_0 I_0 \cos(300t) \times \pi r^2}{L}$$



$\therefore \quad e = -\dfrac{d\phi}{dt} = \dfrac{300\pi\mu_0 I_0 r^2 \sin(300t)}{L}$

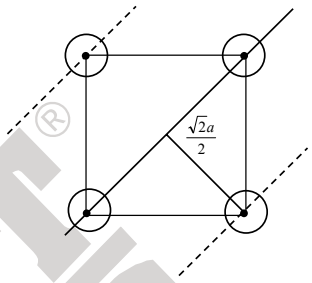
As, $M = iA = \dfrac{300.\pi^2 r^4}{LR}\mu_0 I_0 \sin(300t)$

$M = 6\mu_0 I_0 \sin(300t)$

$\therefore \quad N = 6$

$\therefore$ **The answer is 6**

***41.** Four solid spheres each of diameter $\sqrt{5}$ cm and mass 0.5 kg are placed with their centres at the corners of a square of side 4 cm. The moment of inertia of the system about the diagonal of the square is $N \times 10^{-4}$ kg-m^2, then N is



Sol.: $I = 4 \times \dfrac{2}{5}MR^2 + 2M\left(\dfrac{a}{\sqrt{2}}\right)^2$

$I = 4 \times \dfrac{2}{5} \times 0.5 \times \dfrac{5}{4} \times 10^{-4} + 2 \times 0.5 \times 8 \times 10^{-4}$

$I = 9 \times 10^{-4}$ kg m^2

$\therefore$ **The answer is 9**

42. The activity of a freshly prepared radioactive sample is 10^{10} disintegrations per second, whose mean life is 10^9 s. The mass of an atom of this radioisotope is 10^{-25} kg. The mass (in mg) of the radioactive sample is

Sol.: $-\dfrac{dN}{dt} = \lambda N$

Given, $-\dfrac{dN}{dt} = 10^{10}$, $\lambda = \dfrac{1}{t_{av}} = \dfrac{1}{10^9}$ s^{-1}

$\therefore \quad N = \dfrac{1}{\lambda}\left(-\dfrac{dN}{dt}\right) = 10^{19}$

Total mass = $10^{-25} \times 10^{19}$ kg = 1 mg

$\therefore$ **The answer is 1**

***43.** Steel wire of length 'L' at 40^o C is suspended from the ceiling and then a mass 'm' is hung from its free end. The wire is cooled down from 40^{o}C to 30^{o}C to regain its original length 'L'. The coefficient of linear thermal expansion of the steel is 10^{-5} / ^{o}C, Young's modulus of steel is 10^{11} N/m^2 and radius of the wire is 1 mm. Assume that L >> diameter of the wire. Then the value of 'm in kg is nearly

Sol.: $mg = YA\alpha\Delta T$

$m = \dfrac{\alpha A Y \Delta T}{g} = \dfrac{10^{-5} \times \pi(10^{-3})^2 \times 10^{11} \times 10}{10}$

$m = \pi\ kg = 3.14$ kg

$\therefore$ **The answer is 3**

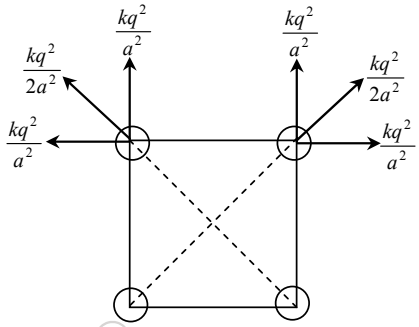
44. Four point charges, each of $+q$, are rigidly fixed at the four corners of a square planar soap film of side 'a'. The surface tension of the soap film is γ. The system of charges and planar film are in equilibrium, and $a = k\left[\frac{q^2}{\gamma}\right]^{1/N}$, where '$k$' is a constant. Then N is

Sol.: $2\frac{kq^2}{a^2} + 2\frac{kq^2}{2\sqrt{2}a^2} = 2\gamma a$

$\Rightarrow \quad a^3 \propto \frac{q^2}{\gamma}$

$a \propto \left(\frac{q^2}{\gamma}\right)^{\frac{1}{3}}$

$\therefore$ **The answer is 3**



***45.** A block is moving on an inclined plane making an angle 45° with the horizontal and the coefficient of friction is μ. The force required to just push it up the inclined plane is 3 times the force required to just prevent it from sliding down. If we define $N = 10\,\mu$, then N is

Sol.: $F_1 = mg(\sin\theta + \mu\cos\theta)$

$F_2 = mg(\sin\theta - \mu\cos\theta)$

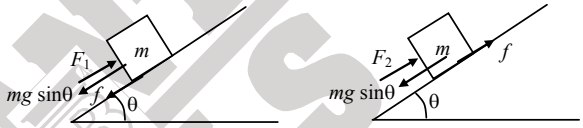
As, $\frac{F_1}{F_2} = 3$

$\Rightarrow \quad \frac{\sin\theta + \mu\cos\theta}{\sin\theta - \mu\cos\theta} = 3$

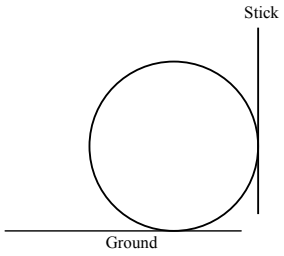
$\Rightarrow \quad \frac{1+\mu}{1-\mu} = 3 \Rightarrow \mu = 1/2$

Now, $N = 10\,\mu = 5$

$\therefore$ **The answer is 5**



***46.** A boy is pushing a ring of mass 2 kg and radius 0.5 m with a stick as shown in the figure. The stick applies a force of 2 N on the ring and rolls it without slipping with an acceleration of 0.3 m/s^2. The coefficient of friction between the ground and the ring is large enough that rolling always occurs and the coefficient of friction between the stick and the ring is (P / 10). The value of P is



Sol.: *FBD* of the ring is shown in figure,

$F - f_1 = ma$ …(i)

$(f_1 - f_2)R = I\alpha$ …(ii)

$a = \alpha R$

From (i)

$f_1 = F - ma = 2 - 0.6 = 1.4$ N

From (ii)

$f_1 - f_2 = \frac{Ia}{R^2} = ma$

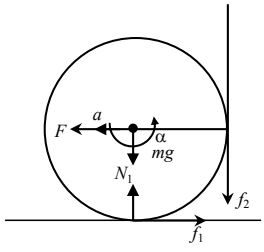
$f_2 = f_1 - ma = 1.4 - 0.6 = 0.8$ N

But $f_2 = \mu F$

$\mu = \frac{f_2}{F} = 0.4$

Now, $P = 10\,\mu = 4$

$\therefore$ **The answer is 4**



SOLUTIONS TO IIT-JEE 2011
MATHEMATICS: Paper-I (Code: 6)

PART – III

SECTION – I (Total Marks: 21)
(Single Correct Answer Type)

This section contains **7 multiple choice questions.** Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

***47.** Let α and β be the roots of $x^2 - 6x - 2 = 0$, with $\alpha > \beta$. If $a_n = \alpha^n - \beta^n$ for $n \geq 1$, then the value of $\frac{a_{10} - 2a_8}{2a_9}$ is

(A) 1 (B) 2 (C) 3 (D) 4

Sol.:

$$\frac{a_{10} - 2a_8}{2a_9}$$

$$= \frac{\alpha^{10} - \beta^{10} - 2(\alpha^8 - \beta^8)}{2(\alpha^9 - \beta^9)}$$

$$= \frac{\alpha^8(\alpha^2 - 2) - \beta^8(\beta^2 - 2)}{2(\alpha^9 - \beta^9)}$$

$$= \frac{\alpha^8.6\alpha - \beta^8.6\beta}{2(\alpha^9 - \beta^9)} \quad \left[\because \alpha^2 - 6\alpha - 2 = 0 \ \& \ \beta^2 - 6\beta - 2 = 0\right]$$

$$= 3$$

Correct choice: (C)

***48.** A straight line L through the point (3, –2) is inclined at an angle 60º to the line $\sqrt{3}x + y = 1$. If L also intersects the x-axis, then the equation of L is

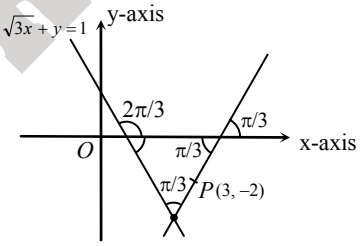
(A) $y + \sqrt{3}x + 2 - 3\sqrt{3} = 0$ (B) $y - \sqrt{3}x + 2 + 3\sqrt{3} = 0$ (C) $\sqrt{3}y - x + 3 + 2\sqrt{3} = 0$ (D) $\sqrt{3}y + x - 3 + 2\sqrt{3} = 0$

Sol.:

Equation will be $y + 2 = \sqrt{3}(x - 3)$

$\Rightarrow \quad y - \sqrt{3}x + 2 + 3\sqrt{3} = 0$

Correct choice: (B)



***49.** Let (x_0, y_0) be the solution of the following equations

$(2x)^{\ln 2} = (3y)^{\ln 3}$

$3^{\ln x} = 2^{\ln y}$.

Then x_0 is

(A) $\frac{1}{6}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) 6

Sol.: Taking log in both equations

$$(\ln 2)^2 + (\ln 2)(\ln x) = (\ln 3)^2 + (\ln 3)(\ln y) \quad \text{..... (i)}$$

and $(\ln 3)(\ln x) = (\ln 2)(\ln y)$ ..... (ii)

From equation (i) and (ii), we get

$$\ln x = -\ln 2 \Rightarrow x = \frac{1}{2}$$

Correct choice: (C)

50. The value of $\int_{\sqrt{\ln 2}}^{\sqrt{\ln 3}} \frac{x \sin x^2}{\sin x^2 + \sin(\ln 6 - x^2)} dx$ is

(A) $\frac{1}{4}\ln\frac{3}{2}$ (B) $\frac{1}{2}\ln\frac{3}{2}$ (C) $\ln\frac{3}{2}$ (D) $\frac{1}{6}\ln\frac{3}{2}$

Sol.: Put $x^2 = t \Rightarrow 2x\ dx = dt$

$$I = \frac{1}{2}\int_{\ln 2}^{\ln 3} \frac{\sin t}{\sin t + \sin(\ln 6 - t)} dt$$

Using property $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

$$\Rightarrow \quad 2I = \frac{1}{2}\int_{\ln 2}^{\ln 3} dt = \frac{1}{2}(\ln 3 - \ln 2)$$

$$\Rightarrow \quad I = \frac{1}{4}\ln\frac{3}{2}.$$

Correct choice: (A)

51. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = \hat{i} - \hat{j} + \hat{k}$ and $\vec{c} = \hat{i} - \hat{j} - \hat{k}$ be three vectors. A vector $\vec{v}$ in the plane of $\vec{a}$ and $\vec{b}$, whose projection on $\vec{c}$ is $\frac{1}{\sqrt{3}}$, is given by

(A) $\hat{i} - 3\hat{j} + 3\hat{k}$ (B) $-3\hat{i} - 3\hat{j} - \hat{k}$ (C) $3\hat{i} - \hat{j} + 3\hat{k}$ (D) $\hat{i} + 3\hat{j} - 3\hat{k}$

Sol.: $\vec{v} = \lambda\vec{a} + \mu\vec{b}$

$= (\lambda + \mu)\hat{i} + \hat{j}(\lambda - \mu) + \hat{k}(\lambda + \mu)$

Now $\frac{\vec{v}.\vec{c}}{|\vec{c}|} = \frac{1}{\sqrt{3}} \Rightarrow (\lambda + \mu) - (\lambda - \mu) - (\lambda + \mu) = 1 \Rightarrow \mu - \lambda = 1$

So $\vec{v} = (1 + 2\lambda)\hat{i} - \hat{j} + (1 + 2\lambda)\hat{k}$

Then take $\lambda = 1$

Correct choice: (C)

***52.** Let $P = \{\theta : \sin\theta - \cos\theta = \sqrt{2}\cos\theta\}$ and $Q = \{\theta : \sin\theta + \cos\theta = \sqrt{2}\sin\theta\}$ be two sets. Then

(A) $P \subset Q$ and $Q - P \neq \varnothing$ (B) $Q \not\subset P$ (C) $P \not\subset Q$ (D) $P = Q$

Sol.: $\sin\theta - \cos\theta = \sqrt{2}\cos\theta \Rightarrow \tan\theta = \sqrt{2} + 1$

and $\sin\theta + \cos\theta = \sqrt{2}\sin\theta \Rightarrow \tan\theta = \frac{1}{\sqrt{2} - 1} = \sqrt{2} + 1$

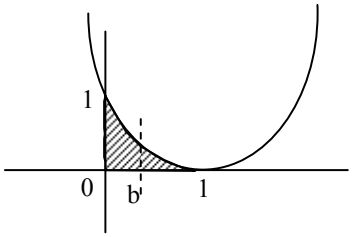
Hence, $P = Q$

Correct choice: (D)

53. Let the straight line $x = b$ divide the area enclosed by $y = (1-x)^2$, $y = 0$ and $x = 0$ into two parts $R_1 (0 \le x \le b)$ and $R_2 (b \le x \le 1)$ such that $R_1 - R_2 = \frac{1}{4}$. Then b equals

(A) $\frac{3}{4}$ (B) $\frac{1}{2}$ (C) $\frac{1}{3}$ (D) $\frac{1}{4}$

Sol.: $\int_0^b (x-1)^2\,dx - \int_b^1 (x-1)^2\,dx = \frac{1}{4}$



$\Rightarrow \quad \frac{(x-1)^3}{3}\Big|_0^b - \frac{(x-1)^3}{3}\Big|_b^1 = \frac{1}{4}$

$\Rightarrow \quad 2\frac{(b-1)^3}{3} = -\frac{1}{12}$

or $\quad b-1 = \frac{-1}{2} \Rightarrow b = \frac{1}{2}$.

Correct choice: (B)

SECTION – II (Total Marks: 16)
(Multiple Correct Answer Type)

This section contains **4 multiple choice questions.** Each question has four choices (A), (B), (C) and (D), out of which **ONE OR MORE** may be correct.

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54. Let M and N be two 3×3 non-singular skew-symmetric matrices such that $MN = NM$. If P^T denotes the transpose of P, then $M^2N^2(M^TN)^{-1}(MN^{-1})^T$ is equal to

(A) M^2 (B) $-N^2$ (C) $-M^2$ (D) MN

Sol.: (The order can not be 3×3 as it will be singular matrix. Solving the problem taking order as $2n\times2n$, where $n \in N$)

$M^2N^2(M^TN)^{-1}(MN^{-1})^T$

$= -MN\ MN(MN)^{-1}(MN^{-1})^T$ (as $MN = NM$)

$= -MN((N^T)^{-1}M^T)$

$= -(MN\ N^{-1}M)$ (as M, N are skew symmetric)

$= -M^2$.

Correct choice: (C)

55. The vector(s) which is/are coplanar with vectors $\hat{i}+\hat{j}+2\hat{k}$ and $\hat{i}+2\hat{j}+\hat{k}$, and perpendicular to the vector $\hat{i}+\hat{j}+\hat{k}$ is/are

(A) $\hat{j}-\hat{k}$ (B) $-\hat{i}+\hat{k}$ (C) $\hat{i}-\hat{j}$ (D) $-\hat{j}+\hat{k}$

Sol.: Required vector

$\vec{d} = \lambda(a\times(b\times c))$, where λ is a real number

$\vec{a} = \hat{i}+\hat{j}+\hat{k}$

$\vec{b} = \hat{i}+\hat{j}+2\hat{k}$

$\vec{c} = \hat{i}+2\hat{j}+\hat{k}$

$\therefore \quad \vec{d} = -4\lambda(\hat{j}-\hat{k})$

Hence, parallel to $(\hat{j}-\hat{k})$

Correct choice: (A), (D)

Alternative

Let $\vec{c}$ is coplanar with vectors $\hat{i}+\hat{j}+2\hat{k}$ and $\hat{i}+2\hat{j}+\hat{k}$.

So $\vec{c} = x(\hat{i}+\hat{j}+2\hat{k}) + y(\hat{i}+2\hat{j}+\hat{k})$

$= (x+y)\hat{i} + (x+2y)\hat{j} + (2x+y)\hat{k}$.

Since, $\vec{c}$ is $\perp$ to vector $\hat{i}+\hat{j}+\hat{k}$,

So $x+y+x+2y+2x+y=0$

$\Rightarrow$ $y=-x$

$\vec{c}=x(\hat{i}+\hat{j}+2\hat{k})-x(\hat{i}+2\hat{j}+\hat{k})$

$=x(-\hat{j}+\hat{k})$

If $x=1$ $=-\hat{j}+\hat{k}$

If $x=-1$ $=\hat{j}-\hat{k}$

Correct choice: (A), (D)

***56.** Let the eccentricity of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ be reciprocal to that of the ellipse $x^2+4y^2=4$. If the hyperbola passes through a focus of the ellipse, then

(A) the equation of the hyperbola is $\frac{x^2}{3}-\frac{y^2}{2}=1$

(B) a focus of the hyperbola is (2, 0)

(C) the eccentricity of the hyperbola is $\sqrt{\frac{5}{3}}$

(D) the equation of the hyperbola is $x^2-3y^2=3$

Sol.: Given equation of hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, eccentricity of ellipse $\frac{x^2}{4}+\frac{y^2}{1}=1$, where $a^2=4,\ b^2=1$

$e=\frac{\sqrt{3}}{2}$, and foci $(\pm\sqrt{3},\ 0)$

So eccentricity of hyperbola $e_1=\frac{2}{\sqrt{3}}$

So $b^2=a^2({e_1}^2-1)$

$b^2=a^2\left(\frac{4}{3}-1\right)=\frac{a^2}{3}$

$a^2=3b^2$

Since hyperbola passes through a focus

$(\pm\sqrt{3},\ 0)$ of ellipse

So $\frac{3}{a^2}-\frac{0}{b^2}=1 \Rightarrow a^2=3$

$\Rightarrow b^2=1$

So equation of hyperbola is $\frac{x^2}{3}-\frac{y^2}{1}=1$

$x^2-3y^2=3$

Foci of hyperbola $(\pm ae,\ 0)$

$\Rightarrow$ $(\pm\sqrt{3}\times\frac{2}{\sqrt{3}},\ 0)$

$\Rightarrow$ $(\pm 2,\ 0)$.

Correct choice: (B), (D)

57. Let $f : R \to R$ be a function such that $f(x+y) = f(x) + f(y),\ \forall x,\ y \in R.$. If $f(x)$ is differentiable at $x = 0,$ then

(A) $f(x)$ is differentiable only in a finite interval containing zero

(B) $f(x)$ is continuous $\forall\ x \in R$

(C) $f'(x)$ is constant $\forall\ x \in R$

(D) $f(x)$ is differentiable except at finitely many points

Sol.: $f : R \to R$

$f(x+y) = f(x) + f(y)\ \forall\ x,\ y \in R$

$x = y = 0 \Rightarrow f(0) = 2f(0) \Rightarrow f(0) = 0$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{f(x) + f(h) - f(x) - f(0)}{h}$$

$$= \lim_{h\to 0} \frac{f(h) - f(0)}{h} \qquad \text{(as } f(x) \text{ is differentiable at } x = 0\text{)}$$

$f'(x) = f'(0) \Rightarrow f(x) = xf'(0) + c$

$f(0) = 0 \Rightarrow c = 0 \Rightarrow f(x) = x\,f'(0)$

$\Rightarrow$ $f'(x)$ is constant $\forall\ x \in R.$

Correct choice: (B), (C)

SECTION – III (Total Marks: 15)
(Paragraph Type)

This section contains **2 paragraphs**. Based upon one of the paragraphs **2 multiple choice questions** and based on the other paragraph **3 multiple choice questions** have to be answered. Each of these questions has four choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

Paragraph for Questions Nos. 58 to 59

Let U_1 and U_2 be two urns such that U_1 contains 3 white and 2 red balls, and U_2 contains only 1 white ball. A fair coin is tossed. If head appears then 1 ball is drawn at random from U_1 and put into U_2. However, if tail appears then 2 balls are drawn at random from U_1 and put into U_2. Now 1 ball is drawn at random from U_2.

58. The probability of the drawn ball from U_2 being white is

(A) $\frac{13}{30}$ (B) $\frac{23}{30}$ (C) $\frac{19}{30}$ (D) $\frac{11}{30}$

Sol.: Required probability

$$\frac{1}{2}\times\left(\frac{{}^3C_1}{{}^5C_1} + \frac{{}^2C_1}{{}^5C_1}\times\frac{1}{2}\right) + \frac{1}{2}\times\left(\frac{{}^3C_2}{{}^5C_2}\times 1 + \frac{{}^2C_2}{{}^5C_2}\times\frac{1}{3} + \frac{{}^3C_1\times{}^2C_1}{{}^5C_2}\times\frac{2}{3}\right)$$

$$= \frac{1}{2}\left\{\frac{3}{5} + \frac{1}{5} + \frac{3}{10} + \frac{1}{30} + \frac{12}{30}\right\}$$

$$= \frac{23}{30}$$

Correct choice: (B)

59. Given that the drawn ball from U_2 is white, the probability that head appeared on the coin is

(A) $\frac{17}{23}$ (B) $\frac{11}{23}$ (C) $\frac{15}{23}$ (D) $\frac{12}{23}$

Sol.: $P\left(\frac{E_1}{A}\right)=\frac{\frac{1}{2}\times\frac{4}{5}}{\frac{1}{2}\times\frac{4}{5}+\frac{1}{2}\times\frac{22}{30}}=\frac{12}{23}$

Correct choice: (D)

Paragraph for Questions Nos. 60 to 62

Let a, b and c be three real numbers satisfying

$$[a\ \ b\ \ c]\begin{bmatrix}1 & 9 & 7\\ 8 & 2 & 7\\ 7 & 3 & 7\end{bmatrix}=[0\ \ 0\ \ 0] \quad \text{.....(E)}$$

60. If the point $P(a,b,c)$, with reference to (E), lies on the plane $2x+y+z=1$, then the value of $7a+b+c$ is

(A) 0 (B) 12 (C) 7 (D) 6

Sol.: As $\begin{vmatrix}1 & 9 & 7\\ 8 & 2 & 7\\ 7 & 3 & 7\end{vmatrix}=0$

$\Rightarrow$ equation E has non trivial solution

$(a,\ b,\ c)\equiv(-\lambda,\ -6\lambda,\ 7\lambda)$

Point satisfies the plane

$-2\lambda-6\lambda+7\lambda=1 \Rightarrow \lambda=-1$

$(a,\ b,\ c)\equiv(1,\ 6,\ -7)$

$\Rightarrow \quad 7a+b+c=6$

Correct choice: (D)

61. Let ω be a solution of $x^3-1=0$ with $\text{Im}(\omega)>0$. If $a=2$ with b and c satisfying (E), then the value of $\frac{3}{\omega^a}+\frac{1}{\omega^b}+\frac{3}{\omega^c}$ is equal to

(A) –2 (B) 2 (C) 3 (D) –3

Sol.: If $a=2\Rightarrow b=12,\ c=-14$

$$\frac{3}{\omega^2}+\frac{1}{\omega^{12}}+\frac{3}{\omega^{-14}}=1+3(\omega+\omega^2)=-2\,.$$

Correct choice: (A)

62. Let $b=6$, with a and c satisfying (E). If α and β are the roots of the quadratic equation $ax^2+bx+c=0$, then $\sum_{n=0}^{\infty}\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)^n$

(A) 6 (B) 7 (C) $\frac{6}{7}$ (D) ∞

Sol.: $b=6\Rightarrow a=1,\ c=-7$

$x^2+6x-7=0$

$\frac{1}{\alpha}+\frac{1}{\beta}=\frac{-6}{-7}=\frac{6}{7}$

$\Rightarrow \quad \sum_{n=0}^{\infty}\left(\frac{6}{7}\right)^n=\frac{1}{1-\frac{6}{7}}=7\,.$

Correct choice: (B)

SECTION – IV (Total Marks: 28)
(Integer Answer Type)

This section contains **7 questions**. The answer to each of the questions is a **single-digit integer**, ranging from 0 to 9. The bubble corresponding to the correct answer is to be darkened in the ORS.

***63.** The minimum value of the sum of real numbers a^{-5}, a^{-4},$3a^{-3}$, 1, a^8 and a^{10} with $a > 0$ is

Sol.: $a^{-5} + a^{-4} + 3a^{-3} + 1 + a^8 + a^{10}$

$\Rightarrow \quad \frac{1}{a^5} + \frac{1}{a^4} + \frac{3}{a^3} + 1 + a^8 + a^{10}$

$\Rightarrow \quad \frac{1}{a^5} + \frac{1}{a^4} + \frac{1}{a^3} + \frac{1}{a^3} + \frac{1}{a^3} + a^8 + a^{10} + 1 \geq 8$ (by using A.M. $\geq$ G.M.))

Ans. 8

64. Let $f(\theta) = \sin\left(\tan^{-1}\left(\frac{\sin\theta}{\sqrt{\cos 2\theta}}\right)\right)$, where $-\frac{\pi}{4} < \theta < \frac{\pi}{4}$. Then the value of $\frac{d}{d(\tan\theta)}(f(\theta))$ is

Sol.: $\tan^{-1}\left(\frac{\sin\theta}{\sqrt{\cos 2\theta}}\right) = \sin^{-1}\left(\frac{\sin\theta}{\sqrt{1-\sin^2\theta}}\right)$

$= \sin^{-1}(\tan\theta)$

$\Rightarrow \quad \sin\left(\tan^{-1}\frac{\sin\theta}{\sqrt{\cos 2\theta}}\right) = \sin(\sin^{-1}(\tan\theta)) = \tan\theta$ as $\theta \in \left(-\frac{\pi}{4}, \frac{\pi}{4}\right)$

$\Rightarrow \quad \frac{d}{d(\tan\theta)}(f(\theta)) = 1$

Ans. 1

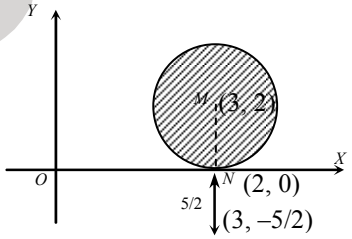
***65.** If z is any complex number satisfying $|z-3-2i| \leq 2$, then the minimum value of $|2z-6+5i|$ is

Sol. $2|z-3+\frac{5}{2}i| = 2|z-3-2i+\frac{9}{2}i| \geq 2\left(||z-3-2i| - |\frac{9i}{2}||\right)$

As $||z-3-2i| - \left|\frac{9}{2}\right|| \geq \left|\frac{5}{2}\right|$

$\Rightarrow \quad 2|z-3+\frac{5}{2}i| \geq 5$

Alternative
It is also clear from the figure



66. Let $f:[1, \infty) \to [2, \infty)$ be a differentiable function such that $f(1) = 2$. If $6\int_1^x f(t)dt = 3x f(x) - x^3$ for all $x \geq 1$, then the value of $f(2)$ is

Sol.: *(To get the correct equation which satisfies $f(1) = 2$, the question should have been $6\int_1^x f(t)dt = 3x f(x) - x^3 - 5$)

Differentiate both sides w.r.t. x
we get

$6f(x) = 3xf'(x) + 3f(x) - 3x^2$

From differential equation

$6y = 3x\frac{dy}{dx} + 3y - 3x^2$

$\Rightarrow \quad 3x\frac{dy}{dx}-3y=3x^2$

$\Rightarrow \quad \frac{dy}{dx}-\frac{y}{x}=x \Rightarrow f(x)=x^2+x$

$\Rightarrow \quad f(2)=4+2=6$

Ans. 6

***67.** The positive integer value of $n>3$ satisfying the equation $\frac{1}{\sin\left(\frac{\pi}{n}\right)}=\frac{1}{\sin\left(\frac{2\pi}{n}\right)}+\frac{1}{\sin\left(\frac{3\pi}{n}\right)}$ is

Sol.: $\frac{1}{\sin\left(\frac{\pi}{n}\right)}=\frac{\sin\left(\frac{2\pi}{n}\right)+\sin\left(\frac{3\pi}{n}\right)}{\sin\left(\frac{2\pi}{n}\right).\sin\left(\frac{3\pi}{n}\right)}$

$\Rightarrow \quad \sin\frac{4\pi}{n}-\sin\frac{3\pi}{n}=0$

$\Rightarrow \quad \frac{7\pi}{2n}=\frac{\pi}{2}$

$n=7$.

Ans. 7

***68.** Let $a_1, a_2, a_3, ..., a_{100}$ be an arithmetic progression with $a_1=3$ and $S_p=\sum_{i=1}^{p}a_i$, $1\le p\le 100$. For any integer n with $1\le n\le 20$, let $m=5n$. If $\frac{S_m}{S_n}$ does not depend on n, then a_2 is

Sol.: $\frac{S_m}{S_n}=\frac{\frac{m}{2}\{2a_1+(m-1)d\}}{\frac{n}{2}\{2a_1+(n-1)d\}}$

$=5\left[\frac{6+(5n-1)d}{6+(n-1)d}\right]$

$=5\left(\frac{6-d+5nd}{6-d+nd}\right)$

This will be independent of n if

$d=6$

$\therefore \quad a_2=3+6=9$

Ans. 9

***69.** Consider the parabola $y^2=8x$. Let Δ_1 be the area of the triangle formed by the end points of its latus rectum and the point $P\left(\frac{1}{2}, 2\right)$ on the parabola, and Δ_2 be the area of the triangle formed by drawing tangents at P and at the end points of the latus rectum. Then $\frac{\Delta_1}{\Delta_2}$ is

Sol.: $\frac{\Delta_1}{\Delta_2}=2$ since the area of the triangle formed by 3 points on a parabola is twice the area of the triangle formed by the tangents at these points.

Ans. 2

BRILLIANT® TUTORIALS

(Code: 2)

SOLUTIONS TO IIT-JEE 2011
Paper-II

Time: 3 Hours **Maximum Marks: 240**

Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose.

INSTRUCTIONS

A. General:

1. The **question paper CODE** is printed on the right hand top corner of this sheet.
2. No additional sheets will be provided for rough work.
3. Blank papers, clipboards, log tables, slide rules, calculators, cellular phones, pagers and electronic gadgets are NOT allowed.
4. Write your name and registration number in the space provided on the back page of this booklet.
5. The answer sheet, a machine-gradable Optical Response Sheet (ORS), is provided separately.
6. DO NOT TAMPER WITH/MULTILATE THE ORS OR THE BOOKLET.
7. Do not break the seals of the question-paper booklet before being instructed to do so by the invigilators.
8. This Question Paper contains 60 questions.
9. On breaking the seals, please check that all the questions are legible.

B. Filling the Right Part of the ORS:

10. The ORS also has a CODE printed on its Left and Right parts.
11. Make sure the CODE on the ORS is the same as that on this booklet. **If the codes do not match, ask for a change of the booklet.**
12. Write your Name, Registration No. and the name of centre and sign **with pen** in the boxes provided. **Do not write them anywhere else.** Darken the appropriate bubble **UNDER** each digit of your registration No. with a **good quality HB pencil.**

C. Question paper format and Marking Scheme:

13. The question paper consists of **3 parts** (Chemistry, Physics and Mathematics). Each part consists of **four sections**.
14. In **Section I** (Total Marks: 24), for each question you will be awarded **3 marks** if you darken **ONLY** the bubble corresponding to the correct answer and **zero marks** if no bubble is darkened. In all other cases, **minus one (–1) mark** will be awarded.
15. In **Section II** (Total Marks: 16), for each question you will be awarded **4 marks** if you darken **ALL** the bubble(s) corresponding to the correct answer(s) **ONLY** and **zero marks** otherwise. There are **no negative marks** in this section.
16. In **Section III** (Total Marks: 24), for each question you will be awarded **4 marks** if you darken **ONLY** the bubble corresponding to the correct answer and **zero marks** otherwise. There are **no negative marks** in this section.
17. In **Section IV** (Total Marks: 16), for each question you will be awarded **2 marks** for each row in which you have darkened **ALL** the bubble(s) corresponding to the correct answer(s) **ONLY** and **zero marks** otherwise. There are **no negative marks** in this section.

SOLUTIONS TO IIT-JEE 2011
CHEMISTRY: Paper-II (Code: 2)

PART – I

Useful Data

$R = 8.314\ JK^{-1}\ mol^{-1}$ or $8.206 \times 10^{-2}\ L\ atm\ K^{-1}\ mol^{-1}$	$1\ F = 96500\ C\ mol^{-1}$
$h = 6.626 \times 10^{-34}\ Js$	$1\ eV = 1.602 \times 10^{-19}\ J$
$c = 3.0 \times 10^{8}\ m\ s^{-1}$	$N_A = 6.022 \times 10^{23}$

SECTION – I (Total Marks: 24)
(Single Correct Answer Type)

This section contains **8 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

Note: Questions with (*) mark are from syllabus of class XI.

1. Among the following complexes (**K**–**P**),
$K_3[Fe(CN)_6]$ (**K**), $[Co(NH_3)_6]Cl_3$ (**L**), $Na_3[Co(oxalate)_3]$ (**M**), $[Ni(H_2O)_6]Cl_2$ (**N**), $K_2[Pt(CN)_4]$ (**O**) and $[Zn(H_2O)_6](NO_3)_2$ (**P**) the diamagnetic complexes are

(A) **K**, **L**, **M**, **N** (B) **K**, **M**, **O**, **P**
(C) **L**, **M**, **O**, **P** (D) **L**, **M**, **N**, **O**

Sol.:

		E.C.	Ligand	Magnetic character
(K)	Fe^{3+}	$3d^5 4s^0$	Strong field ligand	Paramagnetic
(L)	Co^{3+}	$3d^6 4s^0$	Strong field ligand	Diamagnetic
(M)	Co^{3+}	$3d^6 4s^0$	Strong field ligand	Diamagnetic
(N)	Ni^{2+}	$3d^8 4s^0$	Weak field ligand	Paramagnetic
(O)	Pt^{2+}	$5d^8 6s^0$	Strong field ligand	Diamagnetic
(P)	Zn^{2+}	$3d^{10} 4s^0$	Weak field ligand	Diamagnetic

Correct choice: (C)

2. Passing H_2S gas into a mixture of Mn^{2+}, Ni^{2+}, Cu^{2+} and Hg^{2+} ions in an acidified aqueous solution precipitates

(A) CuS and HgS (B) MnS and CuS
(C) MnS and NiS (D) NiS and HgS

Sol.: Cu^{2+} and Hg^{2+} belong to group II. So, on passing H_2S gas, in acidified aqueous solution, they will precipitate as CuS and HgS respectively because their solubility product values are low.
Correct choice: (A)

3. Consider the following cell reaction:

$$2Fe_{(s)} + O_{2(g)} + 4H^{+}_{(aq)} \longrightarrow 2Fe^{2+}_{(aq)} + 2H_2O_{(l)} \ ; \ E^\circ = 1.67\ V$$

At $[Fe^{2+}] = 10^{-3}$ M, $P(O_2) = 0.1$ atm and pH = 3, the cell potential at 25°C is

(A) 1.47 V (B) 1.77 V
(C) 1.87 V (D) 1.57 V

Sol.: $$E_{cell} = 1.67 - \frac{0.059}{4}\log\frac{(10^{-3})^2}{(10^{-3})^4 \times 0.1}$$

$$= 1.67 - \frac{0.059}{4}\log 10^7$$

$$= 1.67 - \frac{0.059 \times 7}{4} = 1.67 - 0.1$$

$$= 1.57\ V.$$

Correct choice: (D)

4. The freezing point (in °C) of a solution containing 0.1 g of $K_3[Fe(CN)_6]$ (molecular weight 329) in 100 g of water ($K_f = 1.86$ K kg mol^{-1}) is

(A) -2.3×10^{-2} (B) -5.7×10^{-2}

(C) -5.7×10^{-3} (D) -1.2×10^{-2}

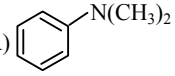
Sol.: $\Delta T_f = i \times K_f \times m$

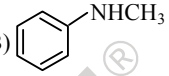
$$\Delta T_f = 4 \times 1.86 \times \frac{0.1 \times 1000}{329 \times 100} = 2.26 \times 10^{-2}$$

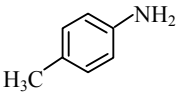
$$\approx 2.3 \times 10^{-2} \Rightarrow T_f = 0 - 2.3 \times 10^{-2} \text{ °C.}$$

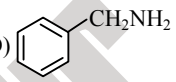
Correct choice: (A)

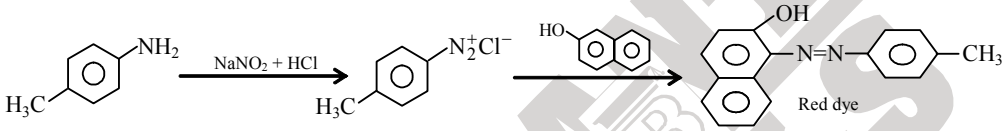
5. Amongst the compounds given, the one that would form a brilliant colored dye on treatment with $NaNO_2$ in dil. HCl followed by addition to an alkaline solution of β-naphthol is

(A) 

(B) 

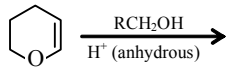
(C) 

(D) 

Sol.: 

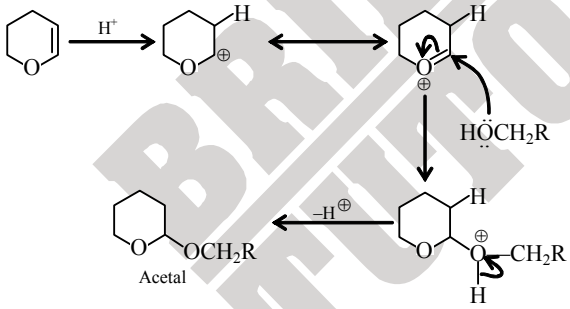
Correct choice: (C)

6. The major product of the following reaction is



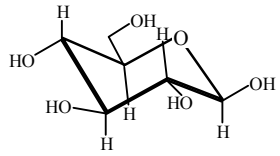
(A) a hemiacetal (B) an acetal

(C) an ether (D) an ester

Sol.: 

Correct choice: (B)

7. The following carbohydrate is



(A) a ketohexose (B) an aldohexose

(C) an α-furanose (D) an α-pyranose

Sol.: Aldohexose

Correct choice: (B)

***8.** Oxidation states of the metal in the minerals haematite and magnetite, respectively, are

(A) (II), (III) in haematite and (III) in magnetite.
(B) (II), (III) in haematite and (II) in magnetite.
(C) (II) in haematite and (II), (III) in magnetite.
(D) (III) in haematite and (II), (III) in magnetite.

Sol.: Haematite: $\overset{III}{Fe_2O_3}$

Magnetite: $Fe_3O_4 \longrightarrow \overset{III}{Fe_2O_3} + \overset{II}{FeO}$

Correct choice: (D)

SECTION – II (Total Marks: 16)
(Multiple Correct Answer(s) Type)

This section contains **4 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONE or MORE** may be correct.

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9. The equilibrium

$$2Cu^{+} \rightleftharpoons Cu^{o} + Cu^{2+}$$

in aqueous medium at 25°C shifts towards the left in the presence of

(A) NO_3^- (B) Cl^- (C) SCN^- (D) CN^-

Sol.: Cuprous compounds can be obtained by passing SO_2 through a solution containing copper sulphate and sodium salt.

$$2CuSO_4 + 2NaCl + SO_2 + 2H_2O \longrightarrow 2CuCl + Na_2SO_4 + 2H_2SO_4$$

$$2CuSO_4 + 2NaCN + SO_2 + 2H_2O \longrightarrow 2CuCN + Na_2SO_4 + 2H_2SO_4$$

$$2CuSO_4 + 2NaCNS + SO_2 + 2H_2O \longrightarrow 2CuCNS + Na_2SO_4 + 2H_2SO_4$$

Correct choice: (B), (C) & (D)

10. For the first order reaction

$$2N_2O_5(g) \longrightarrow 4NO_2(g) + O_2(g)$$

(A) the concentration of the reactant decreases exponentially with time.
(B) the half-life of the reaction decreases with increasing temperature.
(C) the half-life of the reaction depends on the initial concentration of the reactant.
(D) the reaction proceeds to 99.6% completion in eight half-life duration.

Sol.: (A) $[A]_t = [A]_0 e^{-kt}$

$[A]_t \Rightarrow$ concentration after time 't'

$[A]_0$ = initial concentration

(B) Rate const 'k' increases with temperature.

(D) if 99.6% completed

$\Rightarrow$ 0.4% left.

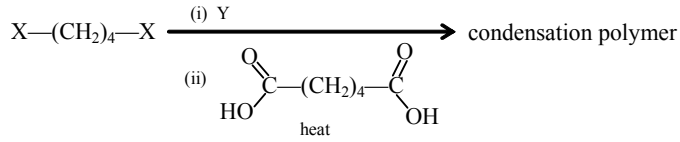
$$k = \frac{2.303}{t} \log \frac{[A]_0}{[A]_t}$$

$$\frac{0.693}{t_{1/2}} = \frac{2.303}{t} \log \frac{[A]_0}{[A]_0 / 256}$$

$$t = 8t_{1/2}$$

Correct choice: (A), (B) & (D)

11. The correct functional group X and the reagent/reaction conditions Y in the following scheme are



(A) X = $COOCH_3$, Y = H_2/Ni/heat
(B) X = $CONH_2$, Y = H_2/Ni/heat
(C) X = $CONH_2$, Y = Br_2/NaOH
(D) X = CN, Y = H_2/Ni/heat

Sol.: (A) $CH_3OOC—(CH_2)_4—COOCH_3 \xrightarrow{H_2/Ni/\Delta} HO—CH_2—(CH_2)_4—CH_2—OH$

(B) $H_2NOC—(CH_2)_4—CONH_2 \xrightarrow{H_2/Ni/\Delta} H_2N—CH_2—(CH_2)_4—CH_2—NH_2$

(C) $H_2NOC—(CH_2)_4—CO—NH_2 \xrightarrow{Br_2/NaOH} H_2N—(CH_2)_4—NH_2$

(D) $NC—(CH_2)_4—CN \xrightarrow{H_2/Ni/\Delta} H_2N—CH_2—(CH_2)_4—CH_2—NH_2$

There is a formation of condensation products with acid and amine or alcohol.

Correct choice: (A), (B), (C) & (D)

12. Reduction of the metal centre in aqueous permanganate ion involves

(A) 3 electrons in neutral medium
(B) 5 electrons in neutral medium
(C) 3 electrons in alkaline medium
(D) 5 electrons in acidic medium

Sol.: In neutral and alkaline media

$$MnO_4^- + 2H_2O + 3e^- \longrightarrow MnO_2 + 4OH^-$$

In acidic medium

$$MnO_4^- + 8H^+ + 5e^- \longrightarrow Mn^{+2} + 4H_2O$$

Correct choice: (A), (C) & (D)

SECTION–III (Total Marks: 24)
(Integer Answer Type)

This section contains **6 questions.** The answer to each of the questions is a **single-digit integer**, ranging from 0 to 9. The bubble corresponding to the correct answer is to be darkened in the ORS.

***13.** The volume (in mL) of 0.1 M $AgNO_3$ required for complete precipitation of chloride ions present in 30 mL of 0.01 M solution of $[Cr(H_2O)_5Cl]Cl_2$, as silver chloride is close to

Sol.: $[Cr(H_2O)_5Cl]Cl_2 \longrightarrow [Cr(H_2O)_5Cl]^{+2} + 2Cl^-$

$\therefore$ m moles of Cl^- $= 30 \times 2 \times 0.01$

On adding $AgNO_3$, m moles of Ag^+ required to precipitate all Cl^- ions is equal to m moles of Cl^-.

$\therefore$ $0.1 \times V = 30 \times 2 \times 0.01$; $\therefore$ $V(AgNO_3) = 6$ ml

$\therefore$ The answer is **6**.

***14.** In 1 L saturated solution of AgCl [K_{sp}(AgCl)=1.6×10^{-10}], 0.1 mol of CuCl [K_{sp}(CuCl) = 1.0×10^{-6}] is added. The resultant concentration of Ag^+ in the solution is 1.6×10^{-x}. The value of "x" is

Sol.: $S_{AgCl} = \sqrt{K_{sp}} = \sqrt{1.6 \times 10^{-10}} = 1.26 \times 10^{-5}$ M

$S_{CuCl} = \sqrt{K_{sp}} = \sqrt{1 \times 10^{-6}} = 1 \times 10^{-3}$ M

$$AgCl \rightleftharpoons \underset{x}{Ag^+} + \underset{\substack{10^{-3} + (1.26 \times 10^{-5}) \\ \approx 10^{-3}}}{Cl^-}$$

$$\therefore \quad [Ag^+] = \frac{K_{sp}(AgCl)}{[Cl^-]} = \frac{1.6 \times 10^{-10}}{10^{-3}}$$

$$= 1.6 \times 10^{-7}$$

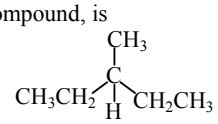
$\therefore$ $x = 7$

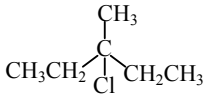
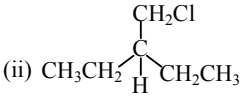
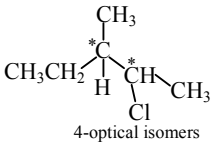
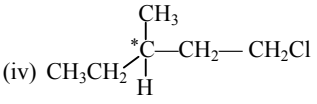
$\therefore$ The answer is **7**.

15. The number of hexagonal faces that are present in a truncated octahedron is

Sol.: The answer is **8**.

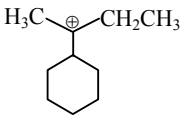
16. The maximum number of isomers (including stereoisomers) that are possible on monochlorination of the following compound, is



Sol.: (i)  (ii)  (iii) 
4-optical isomers
(iv) 
2-optical isomers

∴ The answer is **8**.

***17.** The total number of contributing structures showing hyperconjugation (involving C–H bonds) for the following carbocation is



Sol.: Total number of hyperconjugation structures is one more than the total number of α-H atoms.
∴ The answer is **7**.

18. Among the following, the number of compounds that can react with PCl_5 to give $POCl_3$ is

O_2, CO_2, SO_2, H_2O, H_2SO_4, P_4O_{10}

Sol.: PCl_5 reacts with each of SO_2, H_2O, H_2SO_4 and P_4O_{10} to form $POCl_3$.

$PCl_5 + H_2O$ (insufficient) $\longrightarrow POCl_3 + 2HCl$

$2PCl_5 + H_2SO_4$ or $SO_2(OH)_2 \longrightarrow SO_2Cl_2 + 2POCl_3 + 2HCl$.

$PCl_5 + SO_2 \longrightarrow SOCl_2 + POCl_3$

$6PCl_5 + P_4O_{10} \longrightarrow 10POCl_3$

∴ The answer is **4**.

SECTION – IV (Total Marks : 16)

(Matrix – Match Type)

This section contains **2 questions.** Each question has **four statements** (A, B, C and D) given in **Column I** and **five statements** (p, q, r, s and t) in **Column II.** Any given statement in Column I can have correct matching with **ONE** or **MORE** statement(s) given in Column II. For example, if for a given question, statement B matches with the statements given in q and r, then for the particular question, against statement B, darken the bubbles corresponding to q and r in the ORS.

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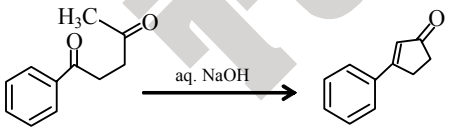
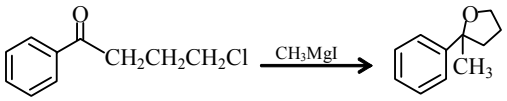
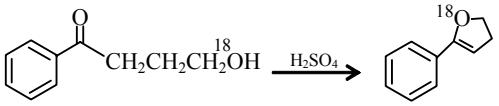
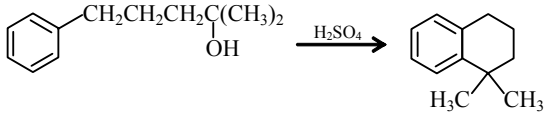
19. Match the reactions in **column I** with appropriate types of steps/reactive intermediate involved in these reactions as given in **column II**.

Column I

(A) 

(B) 

(C) 

(D) 

Column II

(p) Nucleophilic substitution

(q) Electrophilic substitution

(r) Dehydration

(s) Nucleophilic addition

(t) Carbanion

Sol.: **(A) – (r), (s), (t) ; (B) – (p), (s) ; (C) – (r), (s) ; (D) – (q), (r)**

***20.** Match the transformations in **column I** with appropriate options in **column II**.

Column I	**Column II**
(A) $CO_2(s) \longrightarrow CO_2(g)$	(p) phase transition
(B) $CaCO_3(s) \longrightarrow CaO(s) + CO_2(g)$	(q) allotropic change
(C) $2H\bullet \longrightarrow H_2(g)$	(r) ΔH is positive
(D) $P_{(white,solid)} \longrightarrow P_{(red,solid)}$	(s) ΔS is positive
	(t) ΔS is negative

Sol.: **(A) – (p), (r), (s) ; (B) – (p), (r), (s) ; (C) – (t) ; (D) – (q), (t)**

SOLUTIONS TO IIT-JEE 2011
PHYSICS: Paper-II (Code: 2)

PART – II

SECTION – I (Total Marks: 24)
(Single Correct Answer Type)

This section contains **8 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

***21.** A satellite is moving with a constant speed 'V' in a circular orbit about the earth. An object of mass 'm' is ejected from the satellite such that it just escapes from the gravitational pull of the earth. At the time of its ejection, the kinetic energy of the object is

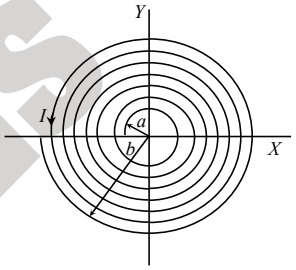
(A) $\frac{1}{2}mV^2$ (B) mV^2 (C) $\frac{3}{2}mV^2$ (D) $2mV^2$

Sol.: To move object to infinity $-\frac{GMm}{r}+K=0$

$K=\frac{GMm}{r}=mV^2 \qquad \left[V=\sqrt{\frac{GM}{r}}\right]$

Correct choice: (B)

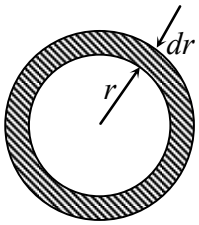
22. A long insulated copper wire is closely wound as a spiral of 'N' turns. The spiral has inner radius 'a' and outer radius 'b'. The spiral lies in the X-Y plane and a steady current 'I' flows through the wire. The Z-component of the magnetic field at the center of the spiral is



(A) $\frac{\mu_0 N I}{2(b-a)}\ln\left(\frac{b}{a}\right)$ (B) $\frac{\mu_0 N I}{2(b-a)}\ln\left(\frac{b+a}{b-a}\right)$

(C) $\frac{\mu_0 N I}{2b}\ln\left(\frac{b}{a}\right)$ (D) $\frac{\mu_0 N I}{2b}\ln\left(\frac{b+a}{b-a}\right)$

Sol.: $dB=\frac{\mu_0\, dNI}{2r}$, $dN=\frac{N}{b-a}dr$



$\Rightarrow \quad dB=\frac{\mu_0 IN}{2(b-a)}\int_a^b\frac{dr}{r} \Rightarrow B=\frac{\mu_0 NI}{2(b-a)}\ln\left(\frac{b}{a}\right)$

Correct choice: (A)

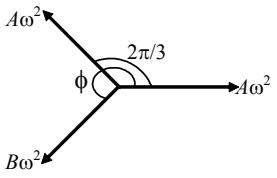
***23.** A point mass is subjected to two simultaneous sinusoidal displacements in x-direction, $x_1(t)=A\sin\omega t$ and $x_2(t)=A\sin\left(\omega t+\frac{2\pi}{3}\right)$. Adding a third sinusoidal displacement $x_3(t)=B\sin(\omega t+\phi)$ brings the mass to a complete rest. The values of B and ϕ are

(A) $\sqrt{2}A, \frac{3\pi}{4}$ (B) $A, \frac{4\pi}{3}$ (C) $\sqrt{3}A, \frac{5\pi}{6}$ (D) $A, \frac{\pi}{3}$

Sol.: $\vec{F}_{net}=0$

$\Rightarrow \quad \vec{a}_1+\vec{a}_2+\vec{a}_3=0$

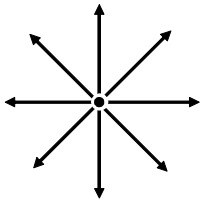
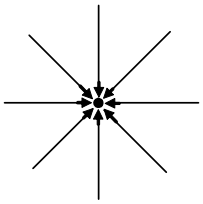
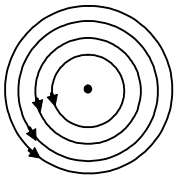
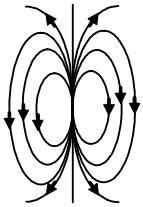
By phasor addition



$\therefore \quad B=A$ and $\phi=\frac{4\pi}{3}$

Correct choice: (B)

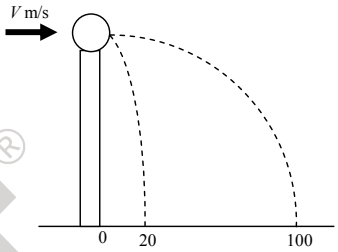
24. Which of the field patterns given below is valid for electric field as well as for magnetic field?

(A)  (B)  (C)  (D) 

Sol.: Induced electric field is nonconservative.

Correct choice: (C)

***25.** A ball of mass 0.2 kg rests on a vertical post of height 5 m. A bullet of mass 0.01 kg, traveling with a velocity V m/s in a horizontal direction, hits the centre of the ball. After the collision, the ball and bullet travel independently. The ball hits the ground at a distance of 20 m and the bullet at a distance of 100 m from the foot of the post. The initial velocity V of the bullet is



(A) 250 m/s (B) $250\sqrt{2}$ m/s

(C) 400 m/s (D) 500 m/s

Sol.: $R = u\sqrt{\dfrac{2h}{g}}$

$\Rightarrow \quad 20 = u_{ball}\sqrt{\dfrac{2\times 5}{10}} \quad \Rightarrow \; u_{ball} = 20$ m/s

$100 = u_{\text{bullet}}\sqrt{\dfrac{2\times 5}{10}} \Rightarrow u_{bullet} = 100$ m/s

Now, by conservation of momentum

$0.01\,V = 0.01\,u_{bullet} + 0.2\,u_{ball} \quad \Rightarrow \quad V = 500$ m/s

Correct choice: (D)

***26.** The density of a solid ball is to be determined in an experiment. The diameter of the ball is measured with a screw gauge, whose pitch is 0.5 mm and there are 50 divisions on the circular scale. The reading on the main scale is 2.5 mm and that on the circular scale is 20 divisions. If the measured mass of the ball has a relative error of 2%, the relative percentage error in the density is

(A) 0.9 % (B) 2.4 % (C) 3.1 % (D) 4.2 %

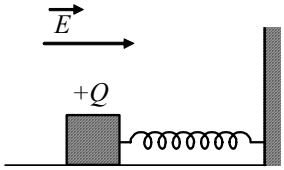
Sol.: $LC = \dfrac{\text{Pitch}}{\text{No. of div. on circular scale}} = \dfrac{0.5}{50} = 0.01$ mm

Reading of diameter = $MSR + LC \times CSR = 2.5 + 0.01\times 20 = 2.70$ mm

$$\frac{\Delta\rho}{\rho}\times 100\% = \frac{\Delta m}{m}\times 100\% + \frac{3\Delta d}{d}\times 100\% \quad = 2\% + 3\times\frac{0.01}{2.70}\times 100\% \quad = 3.1\%$$

Correct choice: (C)

***27.** A wooden block performs SHM on a frictionless surface with frequency, v_0. The block carries a charge $+Q$ on its surface. If now a uniform electric field $\vec{E}$ is switched-on as shown, then the SHM of the block will be



(A) of the same frequency and with shifted mean position

(B) of the same frequency and with the same mean position

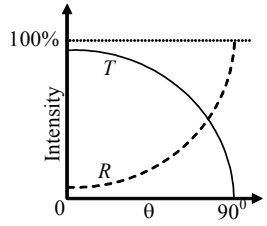
(C) of changed frequency and with shifted mean position

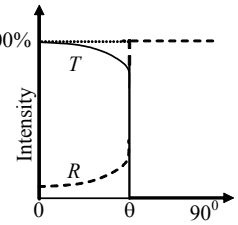
(D) of changed frequency and with the same mean position

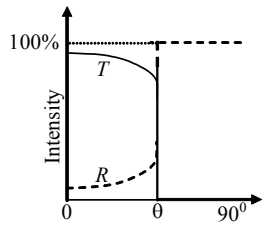
Sol.: Electric field will shift the mean position and frequency will remain same.

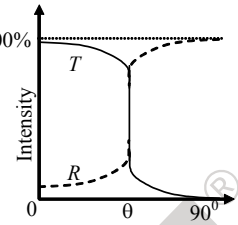
Correct choice: (A)

28. A light ray traveling in glass medium is incident on glass-air interface at an angle of incidence θ. The reflected (R) and transmitted (T) intensities, both as function of θ, are plotted. The correct sketch is

(A) 

(B) 

(C) 

(D) 

Sol.: At θ = 0 partial reflection will occur.

$\therefore\ I_T < 100\%$

At $\theta = \theta_c\ \ I_{\text{transmitted}} = 0$ and $I_{\text{reflected}} = 100\%$

Correct choice: (C)

SECTION – II (Total Marks : 16)
(Multiple Correct Answer(s) Type)

This section contains **4 multiple choice questions.** Each question has four choices (A), (B), (C) and (D) out of which **ONE OR MORE** may be correct.

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29. A series R-C circuit is connected to AC voltage source. Consider two cases; (A) when C is without a dielectric medium and (B) when C is filled with dielectric of constant 4. The current I_R through the resistor and voltage V_C across the capacitor are compared in the two cases. Which of the following is/are true?

(A) $I_R^A > I_R^B$ (B) $I_R^A < I_R^B$ (C) $V_C^A > V_C^B$ (D) $V_C^A < V_C^B$

Sol.: $X_1 = \dfrac{1}{\omega C}, X_2 = \dfrac{1}{4\omega C} \Rightarrow X_2 = \dfrac{X_1}{4}$

$Z_1 = \sqrt{R^2 + X_1^2},\ Z_2 = \sqrt{R^2 + \dfrac{X_1^2}{16}},\ I_{rms} = \dfrac{E_{rms}}{\sqrt{R^2 + X^2}},\ V_C = I_{rms} X_C$

$\therefore\ I_R^B > I_R^A$ and $V_C^A > V_C^B$

Correct choices: (B), (C)

30. Which of the following statement(s) is/are correct?

(A) If the electric field due to a point charge varies as $r^{-2.5}$ instead of r^{-2}, then the Gauss law will still be valid.

(B) The Gauss law can be used to calculate the field distribution around an electric dipole.

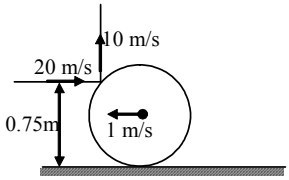
(C) If the electric field between two point charges is zero somewhere, then the sign of the two charges is the same.

(D) The work done by the external force in moving a unit positive charge from point A at potential V_A to point B at potential V_B is $(V_B - V_A)$

Sol.: The work done by the external force in moving a unit positive charge from point A at potential V_A to point B at potential V_B is $(V_B - V_A)$ only when motion of charge particle is without acceleration. Since nothing is given about motion, so option (D) is incorrect.

Correct choice: (C)

***31.** A thin ring of mass 2 kg and radius 0.5 m is rolling without slipping on a horizontal plane with velocity 1 m/s. A small ball of mass 0.1 kg, moving with velocity 20 m/s in the opposite direction, hits the ring at a height of 0.75 m and goes vertically up with velocity 10 m/s. Immediately after the collision



(A) the ring has pure rotation about its stationary CM.

(B) the ring comes to a complete stop.

(C) friction between the ring and the ground is to the left.

(D) there is no friction between the ring and the ground.

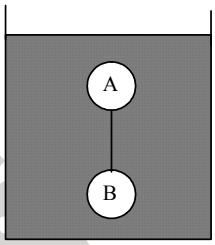
Sol.: Friction between ring and surface is impulsive.

Sense of rotation will not change.

So, friction will act towards left.

Correct choice: (C)

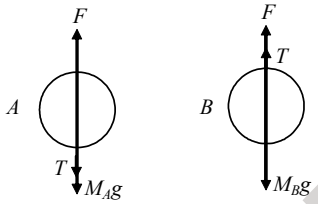
***32.** Two solid spheres A and B of equal volumes but of different densities d_A and d_B are connected by a string. They are fully immersed in a fluid of density d_F. They get arranged into an equilibrium state as shown in the figure with a tension in the string. The arrangement is possible only if



(A) $d_A < d_F$ (B) $d_B > d_F$

(C) $d_A > d_F$ (D) $d_A + d_B = 2d_F$

Sol.:



here F is upthrust force

$$F = T + M_A g \quad \text{.... (i)}$$

$$F + T = M_B g \quad \text{.... (ii)}$$

From (i) & (ii), $F = \dfrac{(M_A + M_B)g}{2}$, as $F = Vd_F g$, $M_A = Vd_A$ and $M_B = Vd_B$

$$\Rightarrow \quad Vd_F g = \frac{V(d_A + d_B)g}{2} \quad \Rightarrow \quad d_F = \frac{d_A + d_B}{2} \quad \text{.... (iii)}$$

Also, $T = \left(\dfrac{M_B - M_A}{2}\right)g$, Here, $T > 0 \Rightarrow \; d_B > d_A$

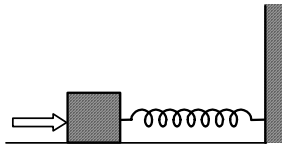
Also, from (iii) $d_B > d_F$ *and* $d_A > d_F$

Correct choices: (A), (B), (D)

SECTION – III (Total Marks : 24)
(Integer Answer Type)

This section contains **6 questions**. The answer to each question is a single-digit integer, ranging from 0 to 9. The bubble corresponding to the correct answer is to be darkened in the ORS.

***33.** A block of mass 0.18 kg is attached to a spring of force-constant 2 N/m. The coefficient of friction between the block and the floor is 0.1. Initially the block is at rest and the spring is un-stretched. An impulse is given to the block as shown in the figure. The block slides a distance of 0.06 m and comes to rest for the first time. The initial velocity of the block in m/s is $V = N/10$. Then N is



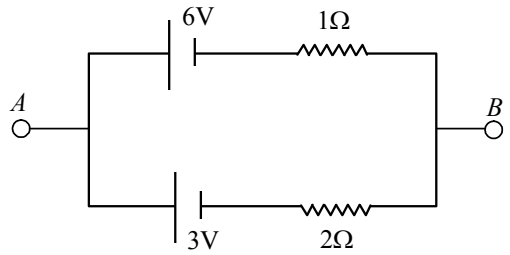
Sol.: $\dfrac{1}{2}mV^2 = \dfrac{1}{2}kx^2 + \mu mg\,x$

$$V = \sqrt{2\mu g\, x + \frac{k\,x^2}{m}} = 0.4 \text{ m/s}$$

$\because N = 10V = 4$

$\therefore$ **The answer is 4.**

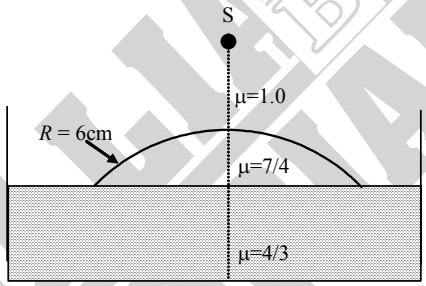
34. Two batteries of different emfs and different internal resistances are connected as shown. The voltage across AB in volts is



Sol.: $V = \dfrac{E_1 r_2 + E_2 r_1}{r_1 + r_2} = 5 \text{ V}$

$\therefore$ **The answer is 5.**

35. Water (with refractive index = $\frac{4}{3}$) in a tank is 18 cm deep. Oil of refractive index $\frac{7}{4}$ lies on water making a convex surface of radius of curvature 'R = 6 cm' as shown. Consider oil to act as a thin lens. An object 'S' is placed 24 cm above water surface. The location of its image is at 'x' cm above the bottom of the tank. Then 'x' is



Sol.: Applying refraction formula at air-oil interface

$$\frac{(7/4)}{v_1} - \frac{1}{-24} = \frac{(7/4)-1}{6} \quad \Rightarrow v_1 = 21 \text{ cm}$$

Again refraction at oil-water interface

$$\frac{(4/3)}{v_2} - \frac{(7/4)}{21} = 0 \Rightarrow \; v_2 = 16 \text{ cm}$$

$\therefore$ From bottom, $x = 18 - 16 = 2$ cm

$\therefore$ **The answer is 2.**

36. A series R-C combination is connected to an AC voltage of angular frequency ω = 500 radian/s. If the impedance of the R-C circuit is $R\sqrt{1.25}$, the time constant (in millisecond) of the circuit is

Sol.: $Z^2 = R^2 + X_c^2 \Rightarrow 1.25\, R^2 = R^2 + X_c^2 \qquad \therefore \; X = 0.5\, R$

Also, $\dfrac{1}{\omega C} = 0.5\, R$

$$RC = \frac{1}{0.5\omega} = 4 \times 10^{-3} \text{ s} = 4 \text{ ms}$$

$\therefore$ **The answer is 4.**

37. A silver sphere of radius 1 cm and work function 4.7 eV is suspended from an insulating thread in free-space. It is under continuous illumination of 200 nm wavelength light. As photoelectrons are emitted, the sphere gets charged and acquires a potential. The maximum number of photoelectrons emitted from the sphere is $A \times 10^Z$ (where $1 < A < 10$). The value of 'Z' is

Sol.: $E = \dfrac{hc}{\lambda} = \dfrac{1240}{200} = 6.2 \text{ eV}, \qquad E = \phi + eV_0 \Rightarrow \; 6.2 - 4.7 = eV_0$

$V_0 = 1.5 \text{ V} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{ne}{r} \;\; \therefore \; n = 1.04 \times 10^7$

This gives, $Z = 7$

$\therefore$ **The answer is 7.**

***38.** A train is moving along a straight line with a constant acceleration 'a'. A boy standing in the train throws a ball forward with a speed of 10 m/s, at an angle of 60° to the horizontal. The boy has to move forward by 1.15 m inside the train to catch the ball back at the initial height. The acceleration of the train, in m/s^2, is

Sol.: $u_{Xbt} = u\cos\theta = 5 \text{ m/s}, \quad a_{Xbt} = -a, \quad X_{bt} = 1.15 \text{ m}$

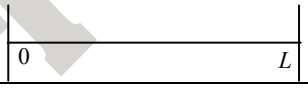
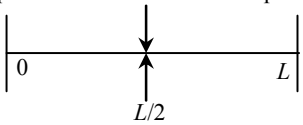
From, $X_{bt} = u_{Xbt}T + \dfrac{1}{2}a_{Xbt}T^2 \qquad \left(T = \dfrac{2u\sin\theta}{g} = \sqrt{3}\ s\right) \qquad \therefore \; a = 5 \text{ m/s}^2$

$\therefore$ **The answer is 5.**

SECTION – IV (Total Marks : 16)
(Matrix-Match Type)

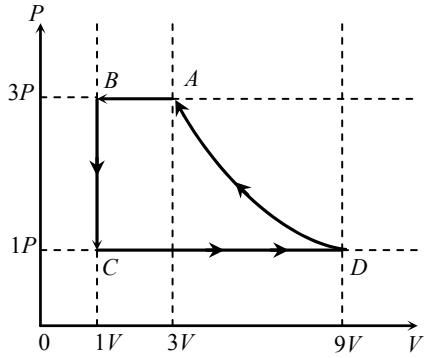
This section contains **2 questions**. Each question has **four statements** (A, B, C and D) given in **Column I** and **five statements** (p, q, r, s and t) in **Column II**. Any given statement in **Column I** can have correct matching with **one or more** statement(s) given in **Column II**. For example, if for a given question, statement B matches with the statements given in q and r, then for the particular question, against statement B, darken the bubbles corresponding to q and r in the ORS.

***39.** **Column I** shows four systems, each of the same length L, for producing standing waves. The lowest possible natural frequency of a system is called its fundamental frequency, whose wavelength is denoted as λ_f. Match each system with statements given in **Column II** describing the nature and wavelength of the standing waves.

Column I	**Column II**
(A) Pipe closed at one end 	**(p)** Longitudinal waves
(B) Pipe open at both ends 	**(q)** Transverse waves
(C) Stretched wire clamped at both ends 	**(r)** $\lambda_f = L$
(D) Stretched wire clamped at both ends and at mid-point 	**(s)** $\lambda_f = 2L$
	(t) $\lambda_f = 4L$

Ans.: **(A) – p, t; (B) – p, s; (C) – q, s; (D) – q, r**

***40.** One mole of a mono atomic ideal gas is taken through a cycle *ABCDA* as shown in the *P-V* diagram. **Column II** gives the characteristics involved in the cycle. Match them with each of the processes given in **Column I.**



Column I	Column II
(A) Process $A \rightarrow B$	**(p)** Internal energy decreases.
(B) Process $B \rightarrow C$	**(q)** Internal energy increases.
(C) Process $C \rightarrow D$	**(r)** Heat is lost.
(D) Process $D \rightarrow A$	**(s)** Heat is gained.
	(t) Work is done on the gas.

Sol.: **(A) – p, r, t; (B) – p, r; (C) – q, s; (D) – r, t**

SOLUTIONS TO IIT-JEE 2011
MATHEMATICS: Paper-II (Code: 2)

PART – III

SECTION – I (Total Marks : 24)
Single Correct Answer Type

This section contains **8 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) for its answer, out of which **ONLY ONE** is correct.

***41.** A value of b for which the equations

$x^2 + bx - 1 = 0$

$x^2 + x + b = 0$,

have one root in common is

(A) $-\sqrt{2}$ (B) $-i\sqrt{3}$ (C) $i\sqrt{5}$ (D) $\sqrt{2}$

Sol.: Let α be the common root

$\Rightarrow \quad \alpha^2 + b\alpha - 1 = 0 \quad \ldots(i)$

And $\quad \alpha^2 + \alpha + b = 0 \quad \ldots(ii)$

Subtracting we get

$$(b-1)\alpha = 1 + b \Rightarrow \alpha = \frac{b+1}{b-1}$$

Substituting the value of α in equation (i)

$$\Rightarrow \quad \left(\frac{b+1}{b-1}\right)^2 + b\left(\frac{b+1}{b-1}\right) - 1 = 0$$

$$\Rightarrow \quad b^3 + 3b = 0 \Rightarrow b = \pm\sqrt{3}i,\ 0$$

Correct choice: (B)

42. Let $\omega \neq 1$ be a cube root of unity and S be the set of all non-singular matrices of the form

$$\begin{bmatrix} 1 & a & b \\ \omega & 1 & c \\ \omega^2 & \omega & 1 \end{bmatrix},$$

where each of a, b and c is either ω or ω^2. Then the number of distinct matrices in the set S is

(A) 2 (B) 6 (C) 4 (D) 8

Sol.: $\det\left(\begin{bmatrix} 1 & a & b \\ \omega & 1 & c \\ \omega^2 & \omega & 1 \end{bmatrix}\right)$

$$= 1 - \omega c + a(\omega^2 c - \omega) + b(\omega^2 - \omega^2)$$

$$= 1 + ac\omega^2 - \omega(a + c)$$

Case I : If $a = \omega$ and $c = \omega \Rightarrow 1 + \omega - 2\omega^2 = -3\omega^2 \neq 0$

Case II : If $a = \omega$ and $c = \omega^2 \Rightarrow 1 + \omega^2 + \omega = 0$

Case III : If $a = \omega^2$ and $c = \omega \Rightarrow 1 + \omega^2 + \omega = 0$

Case IV : If $a = \omega^2$ and $c = \omega^2 \Rightarrow 1 + 1 - 2\omega^3 = 0$

Hence, number of distinct matrices = 2

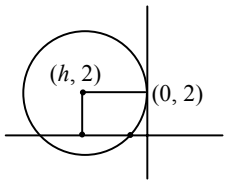
Correct choice: (A)

***43.** The circle passing through the point (–1, 0) and touching the y-axis at (0, 2) also passes through the point

(A) $\left(-\frac{3}{2}, 0\right)$ (B) $\left(-\frac{5}{2}, 2\right)$ (C) $\left(-\frac{3}{2}, \frac{5}{2}\right)$ (D) (–4, 0)

Sol.: $(x-h)^2+(y-2)^2=h^2$

$(h+1)^2+4=h^2 \Rightarrow h=\frac{-5}{2}$



Circle is $\left(x+\frac{5}{2}\right)^2+(y-2)^2=\frac{25}{4}$

Correct choice: (D)

44. If $\lim_{x\to 0}\left[1+x\ln(1+b^2)\right]^{\frac{1}{x}} = 2b\sin^2\theta,\ b>0$ and $\theta\in(-\pi, \pi]$, then the value of θ is

(A) $\pm\frac{\pi}{4}$ (B) $\pm\frac{\pi}{3}$ (C) $\pm\frac{\pi}{6}$ (D) $\pm\frac{\pi}{2}$

Sol.: $\lim_{x\to 0}\left[1+x\ln\left(1+b^2\right)\right]^{\frac{1}{x}}$

$= e^{\lim_{x\to 0} x\ln(1+b^2)\times\frac{1}{x}} = 1+b^2$

$\Rightarrow \quad 1+b^2 = 2b\sin^2\theta$

$\Rightarrow \quad \sin^2\theta = \frac{1+b^2}{2b} = \frac{1}{2}\left(\frac{1}{b}+b\right)$

$\Rightarrow \quad \frac{1}{b}+b \geq 2$

$\Rightarrow \quad \sin^2\theta \geq 1$

$\Rightarrow \quad \sin\theta = \pm 1$

$\Rightarrow \quad \theta = \pm\frac{\pi}{2}$

Correct choice: (D)

45. Let $f:[-1, 2]\to[0, \infty)$ be a continuous function such that $f(x)=f(1-x)$ for all $x\in[-1, 2]$. Let $R_1=\int_{-1}^{2} x f(x)dx$, and R_2 be the area of the region bounded by $y=f(x), x=-1, x=2$, and the x-axis. Then

(A) $R_1=2R_2$ (B) $R_1=3R_2$ (C) $2R_1=R_2$ (D) $3R_1=R_2$

Sol.: $R_1=\int_{-1}^{2} x\ f(x)dx$

$\Rightarrow \quad R_1=\int_{-1}^{2}(1-x)\ f(x)dx$

$\Rightarrow \quad 2R_1=\int_{-1}^{2} f(x)dx,\ R_2=\int_{-1}^{2} f(x)dx$

Hence, $2R_1=R_2$

Correct choice: (C)

46. Let $f(x) = x^2$ and $g(x) = \sin x$ for all $x \in \mathrm{R}$. Then the set of all x satisfying $(f\ o\ g\ o\ g\ o\ f)(x) = (g\ o\ g\ o\ f)(x)$, where $(f\ o\ g)(x) = f(g(x))$, is

(A) $\pm\sqrt{n\pi}$, $n \in \{0, 1, 2, ...\}$ (B) $\pm\sqrt{n\pi}$, $n \in \{1, 2, ...\}$

(C) $\frac{\pi}{2} + 2n\pi$, $n \in \{....,-2, -1, 0, 1, 2,...\}$ (D) $2n\pi$, $n \in \{....,-2, -1, 0, 1, 2,...\}$

Sol.: $\sin^2(\sin x^2) = \sin(\sin x^2)$

$\Rightarrow$ $\sin(\sin x^2) = 0$ or 1

$\Rightarrow$ $\sin x^2 = k\pi$ or $(4k+1)\frac{\pi}{2}$, $k \in$ integer

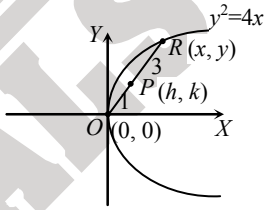
$\Rightarrow$ $\sin x^2 = 0$ only $x^2 = n\pi \Rightarrow x = \pm\sqrt{n\pi}$ where $n \in \{0, 1, 2, ...\}$

Correct choice: (A)

***47.** Let (x, y) be any point on the parabola $y^2 = 4x$. Let P be the point that divides the line segment from (0, 0) to (x, y) in the ratio 1 : 3. Then the locus of P is

(A) $x^2 = y$ (B) $y^2 = 2x$ (C) $y^2 = x$ (D) $x^2 = 2y$

Sol.: Let $P(h, k)$ be the point which divides OR in ratio 1 : 3



$\Rightarrow$ $h = \frac{3.0 + 1.x}{4} \Rightarrow x = 4h$

$k = \frac{3.0 + 1.y}{4} \Rightarrow y = 4k$

Since R lie on the $y^2 = 4x$

$\Rightarrow$ $(4k)^2 = 4(4h)$

$\Rightarrow$ $k^2 = h$

$\Rightarrow$ Locus of P is $y^2 = x$

Correct choice: (C)

***48.** Let $P(6,3)$ be a point on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. If the normal at the point P intersects the x-axis at (9, 0), then the eccentricity of the hyperbola is

(A) $\sqrt{\frac{5}{2}}$ (B) $\sqrt{\frac{3}{2}}$ (C) $\sqrt{2}$ (D) $\sqrt{3}$

Sol.: $\frac{x-6}{6/a^2} = -\frac{y-3}{3/b^2}$

Since normal meet x-axis at (9, 0) so $9 = \frac{6b}{a^2} + 6$

$\therefore$ $\frac{b^2}{a^2} = \frac{1}{2}$

$e = \sqrt{1 + \frac{1}{2}} = \sqrt{\frac{3}{2}}$

Correct choice: (B)

SECTION – II (Total Marks : 16)
Multiple Correct Answer(s) Type

This section contains **4 multiple choice questions.** Each question has four choices (A), (B), (C) and (D) for its answer, out of which **ONE OR MORE** may be correct.

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49. Let E and F be two independent events. The probability that exactly one of them occurs is $\frac{11}{25}$ and the probability of none of them occurring is $\frac{2}{25}$. If $P(T)$ denotes the probability of occurrence of the event T, then

(A) $P(E)=\frac{4}{5}, P(F)=\frac{3}{5}$ (B) $P(E)=\frac{1}{5}, P(F)=\frac{2}{5}$ (C) $P(E)=\frac{2}{5}, P(F)=\frac{1}{5}$ (D) $P(E)=\frac{3}{5}, P(F)=\frac{4}{5}$

Sol.: Let $P(E)=x$ and $P(F)=y$

$$x(1-y)+y(1-x)=\frac{11}{25}$$

$$\Rightarrow \quad x+y-2xy=\frac{11}{25} \qquad \ldots(i)$$

and $$(1-x)(1-y)=\frac{2}{25}$$

$$\Rightarrow \quad x+y-xy=\frac{23}{25} \qquad \ldots(ii)$$

Solving, we get $x=\frac{3}{5}, y=\frac{4}{5}$

or $x=\frac{4}{5}, y=\frac{3}{5}$.

Correct choice: (A), (D)

***50.** Let L be a normal to the parabola $y^2=4x$. If L passes through the point (9, 6), then L is given by

(A) $y-x+3=0$ (B) $y+3x-33=0$ (C) $y+x-15=0$ (D) $y-2x+12=0$

Sol.: Any normal is $y=mx-2m-m^3$

$$\Rightarrow \quad 6=9m-2m-m^3$$

$$\Rightarrow \quad m^3-7m+6=0$$

$$\Rightarrow \quad m=-3,\ 1,\ 2$$

$\therefore$ Normals are $y=-3x+33,\ y=x-3$ and $y=2x-12$

Correct choice: (A), (B), (D)

51. Let $f:(0,1)\to \mathrm{R}$ be defined by $f(x)=\frac{b-x}{1-bx}$, where b is a constant such that $0<b<1$. Then

(A) f is not invertible on (0, 1)

(B) $f\neq f^{-1}$ on (0, 1) and $f'(b)=\frac{1}{f'(0)}$

(C) $f=f^{-1}$ on (0, 1) and $f'(b)=\frac{1}{f'(0)}$

(D) f^{-1} is differentiable on (0, 1)

Sol.: As $f(x)$ is one to one

but its range is $(-1, b)\subset R$

Hence, $f(x)$ is into, therefore $f^{-1}(x)$ does not exists.

Correct choice: (A)

52. If $f(x)=\begin{cases} -x-\frac{\pi}{2}, & x\le -\frac{\pi}{2} \\ -\cos x & -\frac{\pi}{2}<x\le 0 \\ x-1, & 0<x\le 1 \\ \ln x, & x>1 \end{cases}$, then

(A) $f(x)$ is continuous at $x=-\frac{\pi}{2}$

(B) $f(x)$ is not differentiable at $x=0$

(C) $f(x)$ is differentiable at $x=1$

(D) $f(x)$ is differentiable at $x=-\frac{3}{2}$

Sol.: At $x=-\frac{\pi}{2}$: LHL = RHL = 0,

and $f\left(-\frac{\pi}{2}\right)=0 \ \therefore\ f(x)$ is continuous at $x=-\frac{\pi}{2}$

At $x=0$: LHD = $\sin 0 = 0$

RHD = 1 $\therefore\ f(x)$ is not differentiable at $x=0$

At $x=1$: LHD = 1, RHD = 1

$\therefore\ f(x)$ is differentiable at $x=1$

At $x=-\frac{3}{2}$: LHD = RHD = $\sin\left(-\frac{3}{2}\right)$

$\therefore\ f(x)$ is differentiable at $x=\frac{-3}{2}$

Correct choice: (A), (B), (C), (D)

SECTION – III (Total Marks : 24)
(Integer Answer Type)

This section contains **6 questions**. The answer to each question is a **single-digit integer,** ranging from 0 to 9. The correct digit below the question no. in the ORS is to be bubbled.

---

53. The number of distinct real roots of $x^4-4x^3+12x^2+x-1=0$ is

Sol.: Let $f(x)=x^4-4x^3+12x^2+x-1$

$f'(x)=4x^3-12x^2+24x+1$

$f''(x)=12x^2-24x+24$

$=12(x^2-2x+2)>0$

$\Rightarrow$ $f(x)$ has exactly 1 point of minima and $f(0)$ is – 1

$\Rightarrow$ $f(x)$ has two distinct real roots.

Ans. 2

54. Let $y'(x)+y(x)g'(x)=g(x)g'(x),\ y(0)=0,\ x\in \mathrm{R}$, where $f'(x)$ denotes $\frac{d\,f(x)}{dx}$ and $g(x)$ is a given non-constant differentiable function on R with $g(0)=g(2)=0$. Then the value of $y(2)$ is

Sol.: $\frac{dy}{dx}+g'(x)y=g(x)g'(x)$

$y\,e^{g(x)}=\int e^{g(x)}g(x)g'(x)dx$

$= (g(x)-1)e^{g(x)} + c$

$y = g(x) - 1 + c\,e^{-g(x)}$

Put $x = 0$

$0 = -1 + c \Rightarrow c = 1$

Put $x = 2$

$y(2) = -1 + e^{-0} = -1 + 1 = 0$

Ans. 0

55. Let M be a 3 x 3 matrix satisfying $M\begin{bmatrix}0\\1\\0\end{bmatrix} = \begin{bmatrix}-1\\2\\3\end{bmatrix}$, $M\begin{bmatrix}1\\-1\\0\end{bmatrix} = \begin{bmatrix}1\\1\\-1\end{bmatrix}$, and $M\begin{bmatrix}1\\1\\1\end{bmatrix} = \begin{bmatrix}0\\0\\12\end{bmatrix}$. Then the sum of the diagonal entries of M is

Sol.: Let M be $\begin{bmatrix}a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\\ c_1 & c_2 & c_3\end{bmatrix}$

$\therefore \quad M\begin{bmatrix}0\\1\\0\end{bmatrix} = \begin{bmatrix}-1\\2\\3\end{bmatrix}$

$\Rightarrow \quad \left.\begin{aligned} a_2 &= -1\\ b_2 &= 2\\ c_2 &= 3\end{aligned}\right\}$ …(i)

Again, $M\begin{bmatrix}1\\-1\\0\end{bmatrix} = \begin{bmatrix}1\\1\\-1\end{bmatrix}$

$\left.\begin{aligned} a_1 - a_2 &= 1\\ b_1 - b_2 &= 1\\ c_1 - c_2 &= -1\end{aligned}\right\}$

$\Rightarrow \quad a_1 = 1 + a_2 = 1 + (-1) = 0$

$b_1 = 1 + b_2 = 1 + 2 = 3$

$c_1 = -1 + c_2 = -1 + 3 = 2$

Also $c_1 + c_2 + c_3 = 12$

$\Rightarrow \quad c_3 = 12 - c_1 - c_2 = 12 - 2 - 3 = 7$

$\therefore$ Required sum is

$= a_1 + b_2 + c_3$

$= 0 + 2 + 7$

$= 9$

Ans. 9

56. Let $\vec{a} = -\hat{i} - \hat{k},\ \vec{b} = -\hat{i} + \hat{j}$ and $\vec{c} = \hat{i} + 2\hat{j} + 3\hat{k}$ be three given vectors. If $\vec{r}$ is a vector such that $\vec{r} \times \vec{b} = \vec{c} \times \vec{b}$ and $\vec{r}\,.\,\vec{a} = 0$, then the value of $\vec{r}\,.\,\vec{b}$ is

Sol.: $\vec{a} = -\hat{i} - \hat{k},\ \vec{b} = -\hat{i} + \hat{j},\ \vec{c} = \hat{i} + 2\hat{j} + 3\hat{k}$

$\vec{r} \times \vec{b} - \vec{c} \times \vec{b} = 0$

$(\vec{r} - \vec{c}) \times \vec{b} = 0$

$\Rightarrow$ So $\vec{r}-\vec{c} \parallel \vec{b}$

So $\vec{r}-\vec{c}=\lambda\vec{b}$

$\vec{r}=\vec{c}+\lambda\vec{b}=(\hat{i}+2\hat{j}+3\hat{k})+\lambda(-\hat{i}+\hat{j})$

$\hat{r}=(1-\lambda)\hat{i}+(2+\lambda)\hat{j}+3\hat{k}$

$\vec{r}.\vec{a}=0$

$\Rightarrow$ $-(1-\lambda)+0-1\times 3=0$

$\Rightarrow$ $-1+\lambda-3=0,\ \lambda=4$

$\vec{r}=(1-4)\hat{i}+(2+4)\hat{j}+3\hat{k}$

$\vec{r}=-3\hat{i}+6\hat{j}+3\hat{k}$

$\vec{r}.\vec{b}=(-3\hat{i}+6\hat{j}+3\hat{k}).(-\hat{i}+\hat{j})$

$=3+6=9$

So $\vec{r}.\vec{b}=9$

Ans. 9

***57.** The straight line $2x-3y=1$ divides the circular region $x^2+y^2\le 6$ into two parts. If $S=\left\{\left(2,\frac{3}{4}\right),\left(\frac{5}{2},\frac{3}{4}\right),\left(\frac{1}{4},-\frac{1}{4}\right),\left(\frac{1}{8},\frac{1}{4}\right)\right\}$, then the number of point(s) in S lying inside the smaller part is

Sol.: Only points

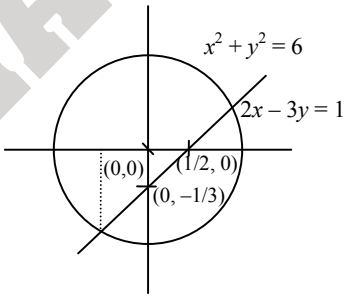
$\left(2,\frac{3}{4}\right),\left(\frac{1}{4},-\frac{1}{4}\right)$ and $\left(\frac{1}{8},\frac{1}{4}\right)$ lies inside the circle

Put value of $O(0,0)$ in equation $2x-3y-1=0$

So $=2\times 0-3\times 0-1=-\text{ve}$

Since points are in smaller region.

So points should be opposite to origin w.r.t. line $2x-3y-1=0$



Case I $\left(2,\frac{3}{4}\right)$

$=2\times 2-3\times\frac{3}{4}-1=3-\frac{9}{4}>0\ +\text{ve}$

Case II $\left(\frac{1}{4},-\frac{1}{4}\right)$

$=2\times\frac{1}{4}-3\times-\frac{1}{4}-1$

$=\frac{2}{4}+\frac{3}{4}-1=+\text{ve}$

Case III $\left(\frac{1}{8},\frac{1}{4}\right)$

$=2\times\frac{1}{8}-3\times\frac{1}{4}-1$

$=\frac{2}{8}-\frac{3}{4}-1$

$=\frac{2-6-8}{8}=-\text{ve}$

Out of these 3 points only $\left(2, \frac{3}{4}\right), \left(\frac{1}{4}, -\frac{1}{4}\right)$ lie inside the smaller part as origin (0, 0) and these points lie on opposite side of line $2x-3y-1=0$

Ans. 2

***58.** Let $\omega = e^{i\pi/3}$ and, *a, b, c, x, y, z* be non-zero complex numbers such that

$a+b+c=x$

$a+b\omega+c\omega^2=y$

$a+b\omega^2+c\omega=z$.

Then the value of $\frac{|x|^2+|y|^2+|z|^2}{|a|^2+|b|^2+|c|^2}$ is

***Sol.:** Value of this expression does not come out to be constant as it depends on choice of *a*, *b*, *c*.

(For example if $a=b=c,\ x=3,\ y=z=1+\sqrt{3}i$ then expression is equal to $\frac{17}{3}$.

Similarly $a=-1,\ b=1,\ c=1,\ x=1,\ y=-1+\sqrt{3}i,\ z=-1+\sqrt{3}i$ then expression is equal to 3 etc.)

But , If we take $\omega = e^{\frac{i2\pi}{3}}$ then it is constant and equal to 3.

SECTION – IV (Total Marks : 16)
(Matrix-Match Type)

This section contains **2 questions**. Each question has four statements (A, B, C and D) given in **Column I** and **five** statements (p, q, r, s and t) in **Column II.** Any given statement in **Column I** can have correct matching with one or more statement(s) given in **Column II.** For example, if for a given question, statement B matches with the statements given in q and r, then for that particular question, against statement B, darken the bubbles corresponding to q and r in the ORS

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59. Match the statements given in **Column –I** with the intervals/union of intervals given in **Column-II.**

Column I	Column II
***(A)** The set $\left\{\text{Re}\left(\frac{2iz}{1-z^2}\right): z \text{ is a complex number}, \lvert z\rvert=1,\ z\neq\pm1\right\}$ is	**(p)** $(-\infty,-1)\cup(1,\infty)$
(B) The domain of the function $f(x)=\sin^{-1}\left(\frac{8(3)^{x-2}}{1-3^{2(x-1)}}\right)$ is	**(q)** $(-\infty,0)\cup(0,\infty)$
***(C)** If $f(\theta)=\begin{vmatrix}1 & \tan\theta & 1\\ -\tan\theta & 1 & \tan\theta\\ -1 & -\tan\theta & 1\end{vmatrix}$, then the set $\left\{f(\theta): 0\le\theta<\frac{\pi}{2}\right\}$ is	**(r)** $[2,\infty)$
(D) If $f(x)=x^{3/2}(3x-10),\ x\ge0$, then $f(x)$ is increasing in	**(s)** $(-\infty,-1]\cup[1,\infty)$
	(t) $(-\infty,0]\cup[2,\infty)$

Sol.: **(A) – (s); (B) – (t); (C) – (r); (D) – (r).**

(A) $\text{Re}\left(\frac{2iz}{1-z^2}\right)=\text{Re}\left(\frac{2i}{\frac{1}{z}-z}\right)$

$$= \text{Re}\left(\frac{2i}{\bar{z}-z}\right) = \text{Re}\left(\frac{2i}{-2i\sin\theta}\right) = -\operatorname{cosec}\theta$$

Real part $\in (-\infty, -1]\cup[1, \infty)$

(B) $\quad -1 \le \dfrac{8.3^x/9}{1-\dfrac{3^{2x}}{9}} \le 1 \qquad -1 \le \dfrac{8.3^x}{9-3^{2x}} \le 1$

Case I: $\dfrac{8.3^x}{9-3^{2x}} + 1 \ge 0$

$$\frac{8.3^x + 9 - 3^{2x}}{(3+3^x)(3-3^x)} \ge 0$$

$$\frac{(3^{2x} - 8.3^x - 9)}{(3+3^x)(3^x-3)} \ge 0$$

$$\frac{3^{2x} - 9.3^x + 3^x - 9}{3^x - 3} \ge 0$$

$$\frac{3^x(3^x-9) + (3^x-9)}{3^x-3} \ge 0$$

$x \in (-\infty, 1)\cup[2, \infty)$ …(i)

$$\frac{8.3^x}{9-3^{2x}} - 1 \le 0$$

Case II: $\dfrac{8.3^x - 9 + 3^{2x}}{9-3^{2x}} \le 0$

$$\frac{3^{2x} + 83^x - 9}{(3+3^x)(3-3^x)} \le 0$$

$$\frac{3^{2x} + 9.3^x - 3^x - 9}{3-3^x} \le 0$$

$$\frac{(3^x-1)(3^x+9)}{3-3^x} \le 0$$

$x \in (-\infty, 0]\cup(1, \infty)$ …(ii)

So common solution is $x \in (-\infty, 0]\cup[2, \infty)$

(C) $\quad f(\theta) = 1(1+\tan^2\theta) + \tan\theta(0) + 1(1+\tan^2\theta)$

$= 2\sec^2\theta$

≥ 2

So solution is $[2, \infty)$

(D) $\quad f(x) = (x)^{3/2}(3x-10),\ x \ge 0$

$f(x) = 3.x^{5/2} - 10x^{3/2}$

$$f'(x) = 3\times\frac{5}{2}x^{3/2} - 10\times\frac{3}{2}x^{1/2} = \frac{15}{2}x^{3/2} - \frac{30}{2}x^{1/2}$$

Since $f(x)$ is an increasing function $f'(x) \ge 0$

$$\Rightarrow \quad \frac{15}{2}\sqrt{x}(x-2) \ge 0 \Rightarrow (x-2) \ge 0 \Rightarrow x \ge 2$$

$x \in [2, \infty)$

60. Match the statements given in **Column –I** with the values in **Column-II.**

Column I		Column II	
(A)	If $\vec{a}=\hat{j}+\sqrt{3}\hat{k}, \vec{b}=-\hat{j}+\sqrt{3}\hat{k}$ and $\vec{c}=2\sqrt{3}\hat{k}$ form a triangle, then the internal angle of the triangle between $\vec{a}$ and $\vec{b}$ is	**(p)**	$\frac{\pi}{6}$
(B)	If $\int_a^b (f(x)-3x)\,dx = a^2-b^2$, then the value of $f\left(\frac{\pi}{6}\right)$ is	**(q)**	$\frac{2\pi}{3}$
(C)	The value of $\frac{\pi^2}{\ln 3}\int_{7/6}^{5/6}\sec(\pi x)\,dx$ is	**(r)**	$\frac{\pi}{3}$
***(D)**	The maximum value of $\left\lvert Arg\left(\frac{1}{1-z}\right)\right\rvert$ for $\lvert z\rvert=1, z\neq 1$ is given by	**(s)**	π
		(t)	$\frac{\pi}{2}$

Sol.: **(A) – (q); (B) – (p); (C) – (s); (D) – (t).**

(A) As $\vec{a}+\vec{b}=\vec{c}$

So angle between $\vec{a}$ and $\vec{b}$ is

$$(\hat{j}+\sqrt{3}\hat{k}).(\hat{j}-\sqrt{3}\hat{k}) = \left|(\hat{j}+\sqrt{3}\hat{k})\right|\left|(\hat{j}-\sqrt{3}\hat{k})\right|\cos\theta$$

$$\cos\theta = -\frac{1}{2} \Rightarrow \theta = \frac{2\pi}{3}$$

(B) $$\int_a^b f(x)dx = \frac{b^2-a^2}{2} \Rightarrow f(x)=x$$

$$f\left(\frac{\pi}{6}\right)=\frac{\pi}{6}$$

(C) Let $x=t+1 \Rightarrow \frac{\pi^2}{\ln 3}\int_{1/6}^{-1/6}\sec(\pi+\pi t)dt$

$$=\frac{2\pi^2}{\ln 3}\int_0^{1/6}\sec\pi t\,dt = \frac{2\pi}{\ln 3}\left|\ln(\sec\pi t+\tan\pi t)\right|_0^{1/6}=\pi$$

(D) Let $z=e^{i\theta}$

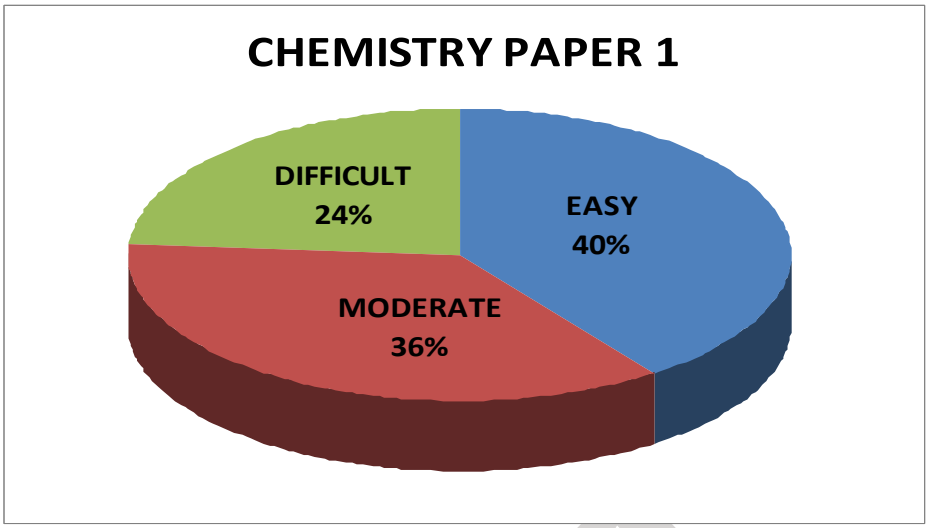
$$\Rightarrow \quad 1-z=1-\cos\theta-i\sin\theta \;=2\sin\frac{\theta}{2}\left(\sin\frac{\theta}{2}-i\cos\frac{\theta}{2}\right)$$

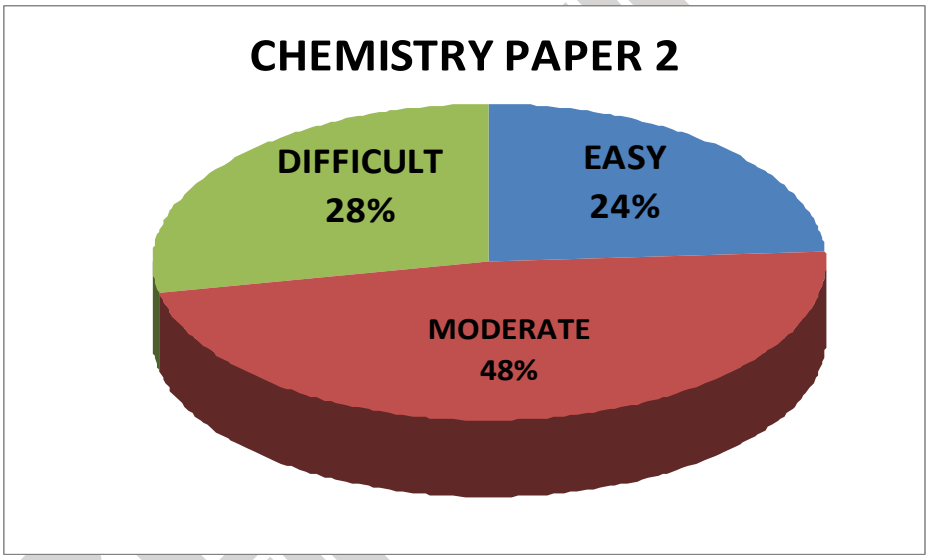
$$\Rightarrow \quad \frac{1}{1-z}=\frac{1}{2\sin\frac{\theta}{2}\left(\sin\frac{\theta}{2}-i\cos\frac{\theta}{2}\right)}=\frac{\sin\frac{\theta}{2}+i\cos\frac{\theta}{2}}{2\sin\frac{\theta}{2}} \qquad =\frac{1}{2}+\frac{i}{2}\cot\frac{\theta}{2}$$

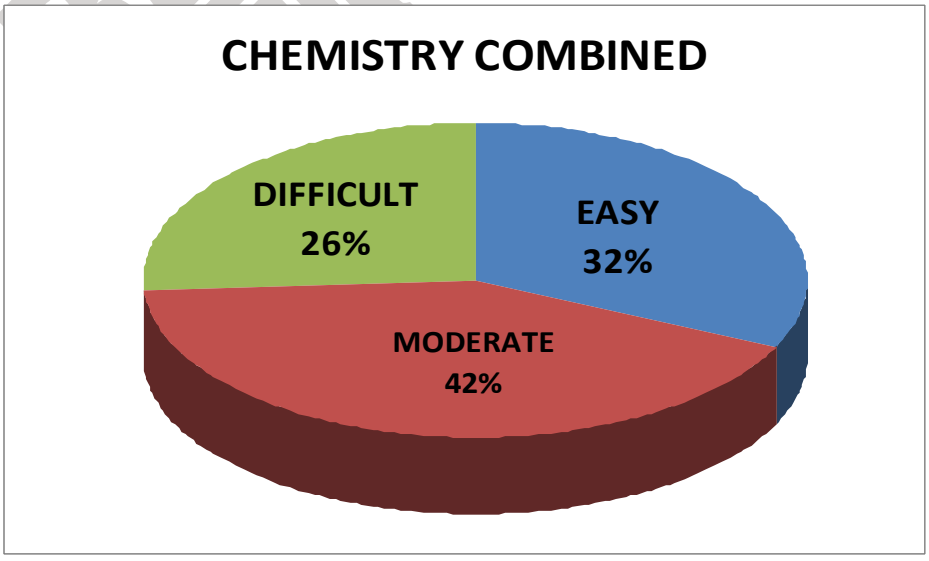
$\therefore \quad \frac{1}{1-z}$ lies on $x=\frac{1}{2}$

$\Rightarrow \quad \left|\arg\left(\frac{1}{1-z}\right)\right|$ tends towards $\frac{\pi}{2}$ as $\cot\frac{\theta}{2}\to\pm\infty$.

BREAK UP 1 (LEVEL OF DIFFICULTY)







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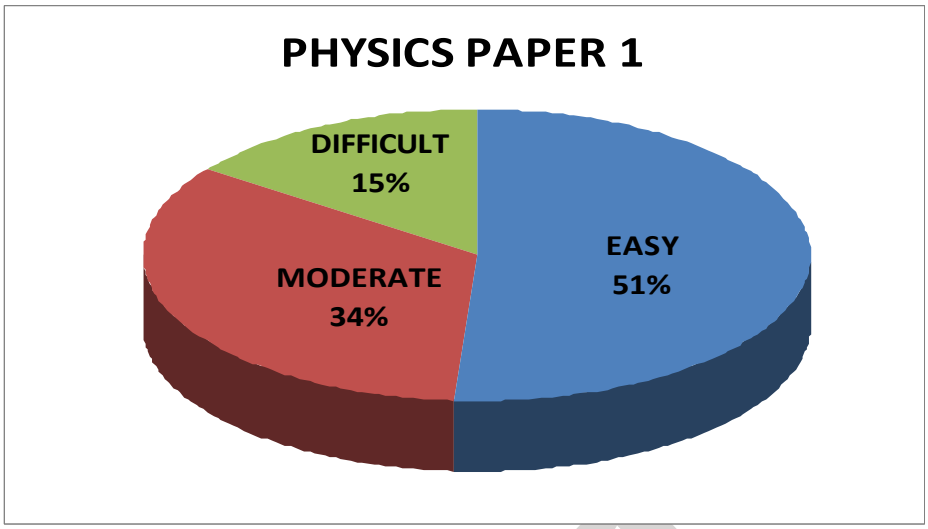

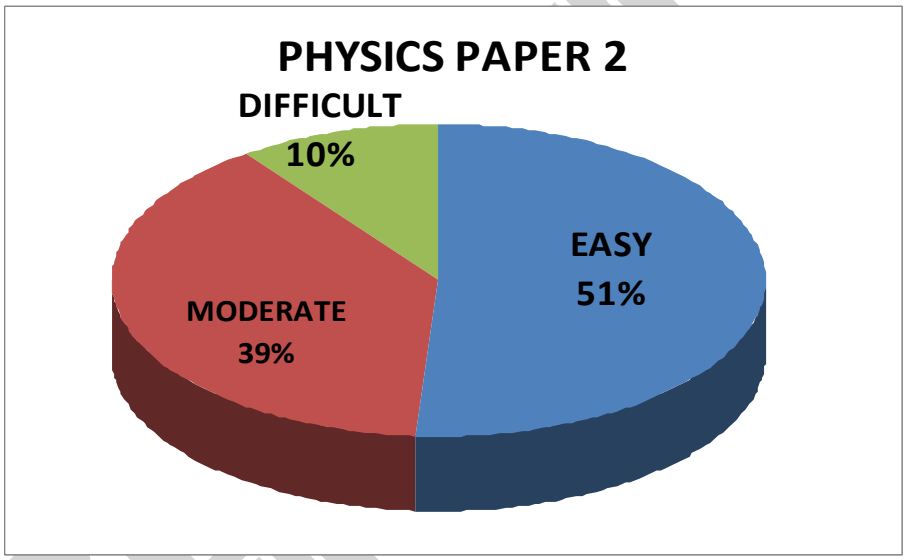

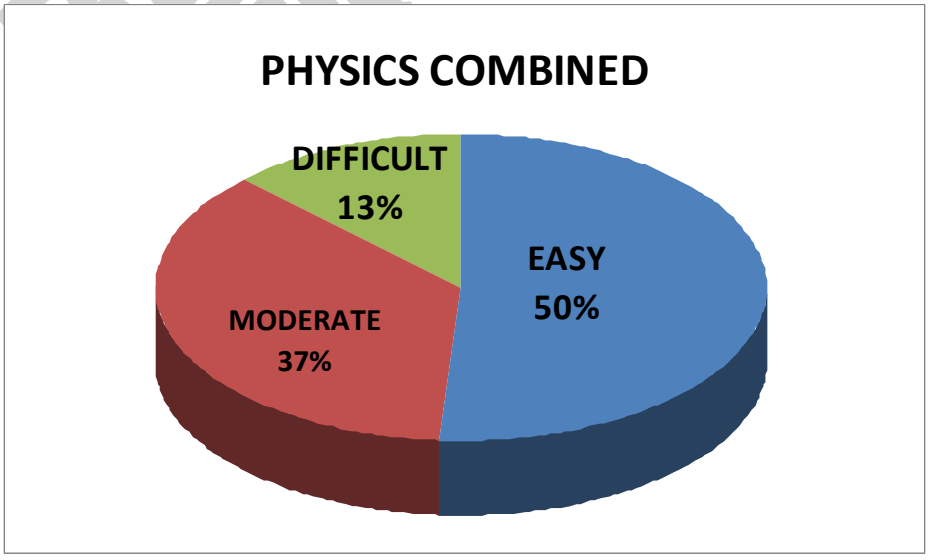

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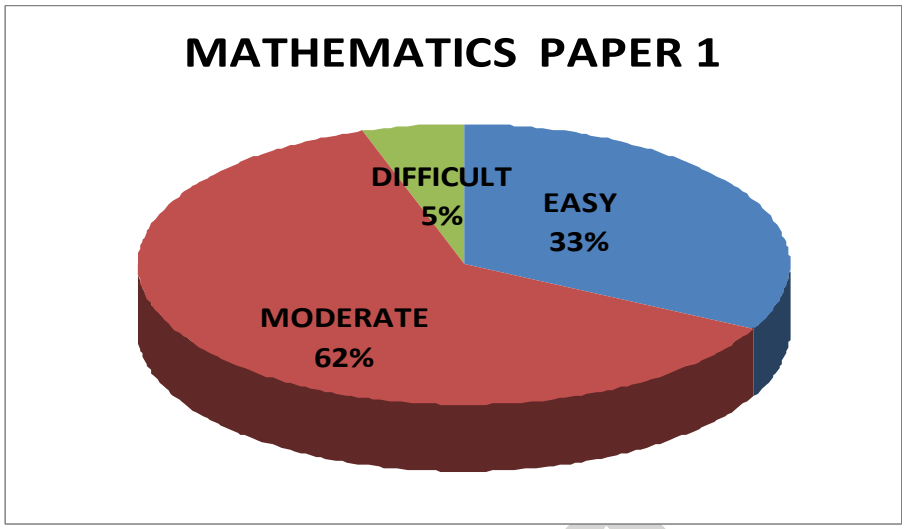
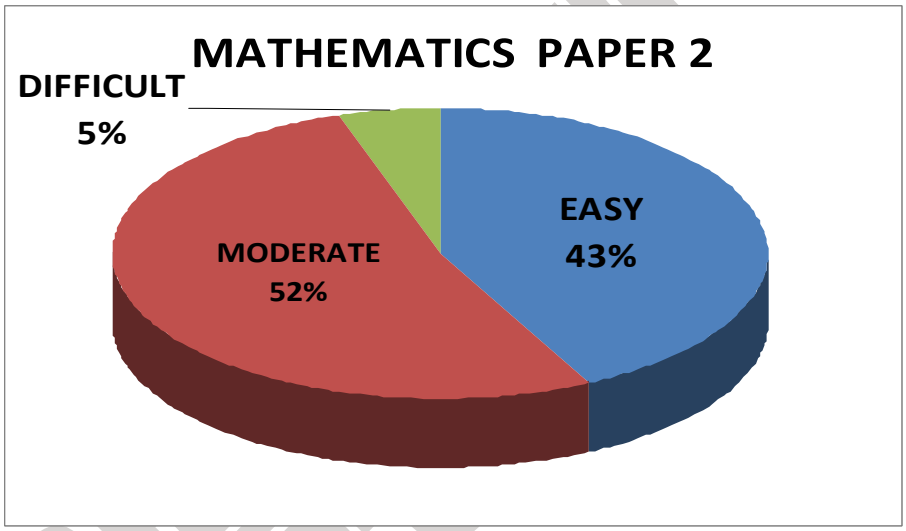
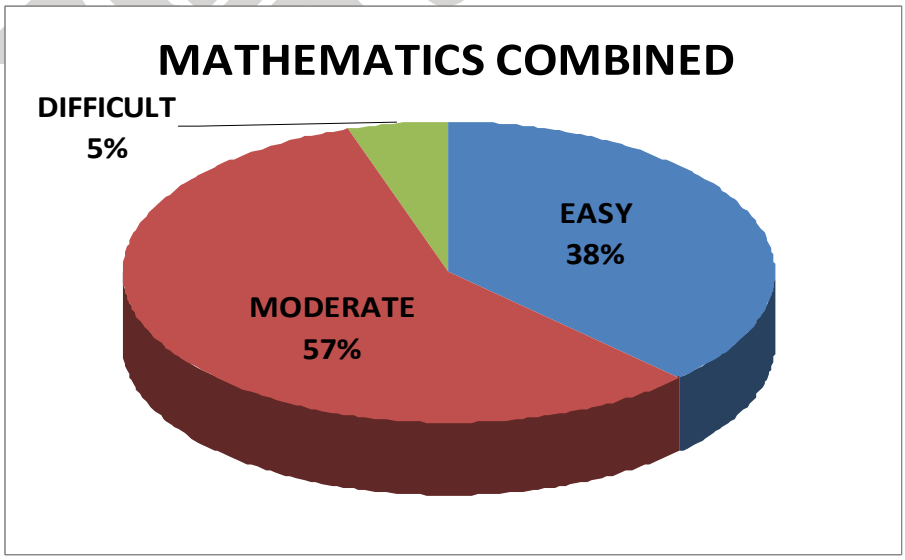
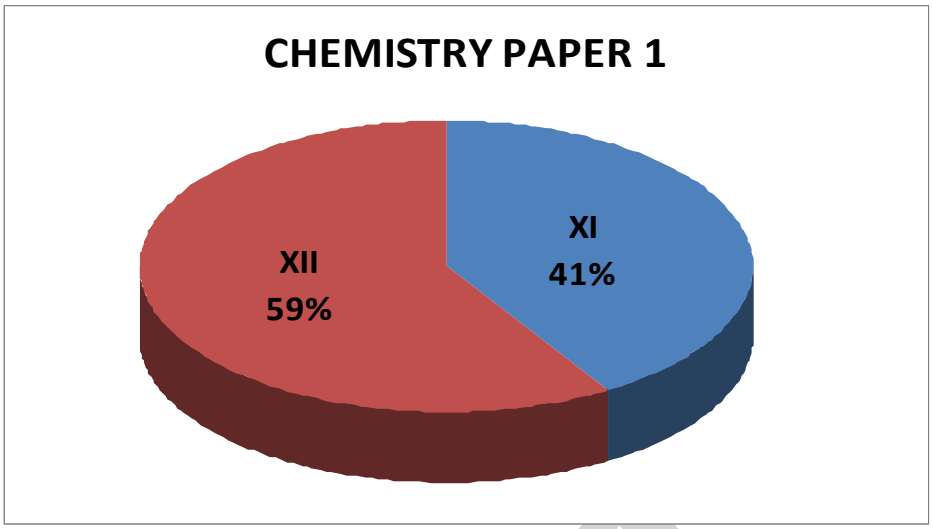
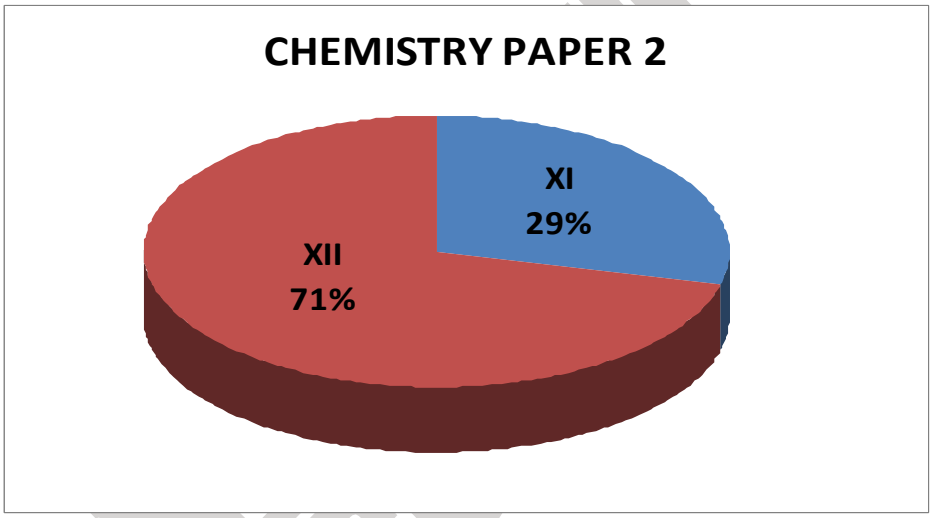


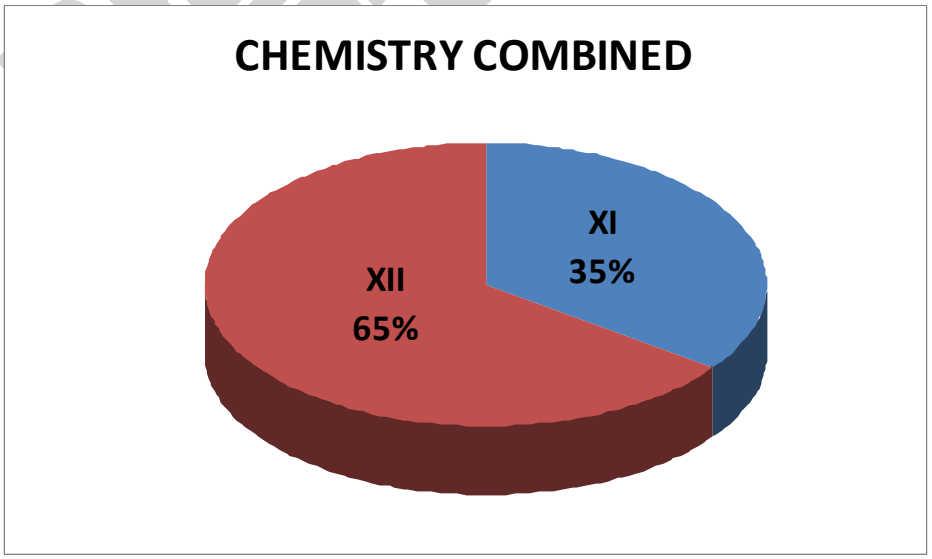




BREAK UP 2 (XI-XII)







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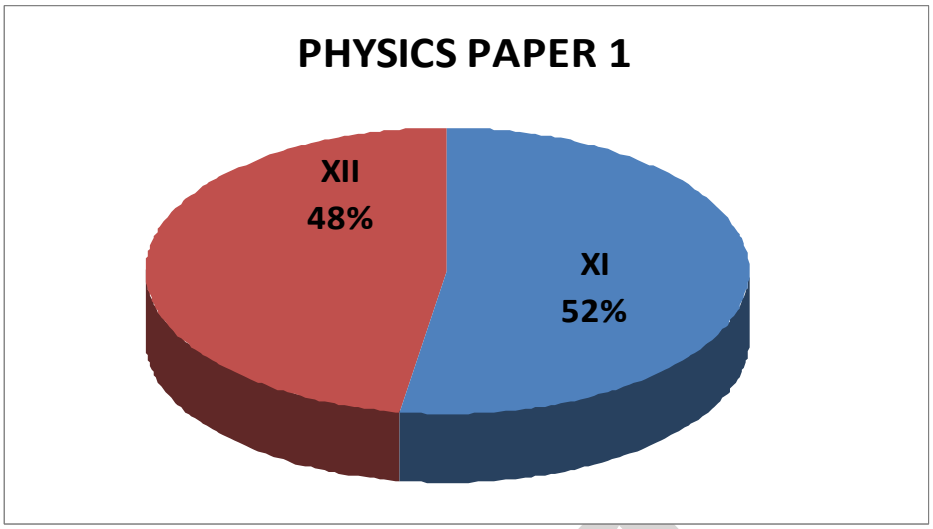
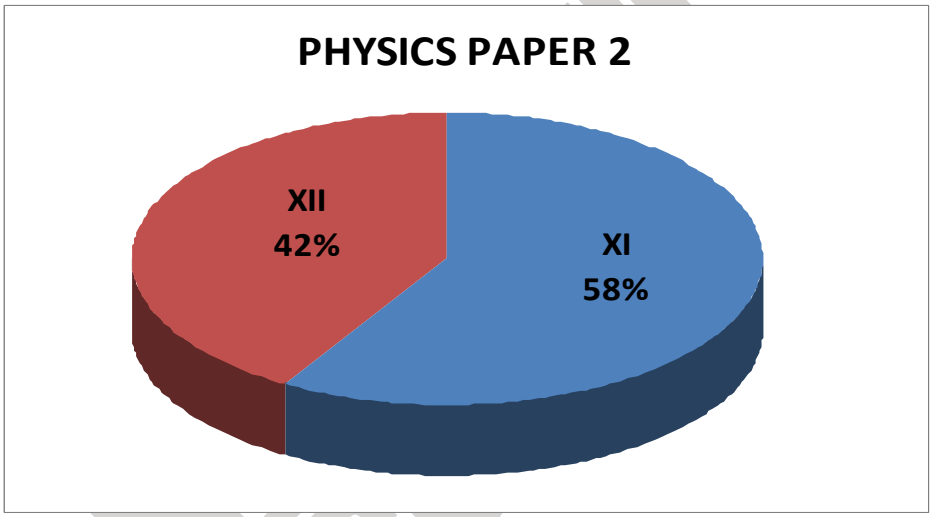
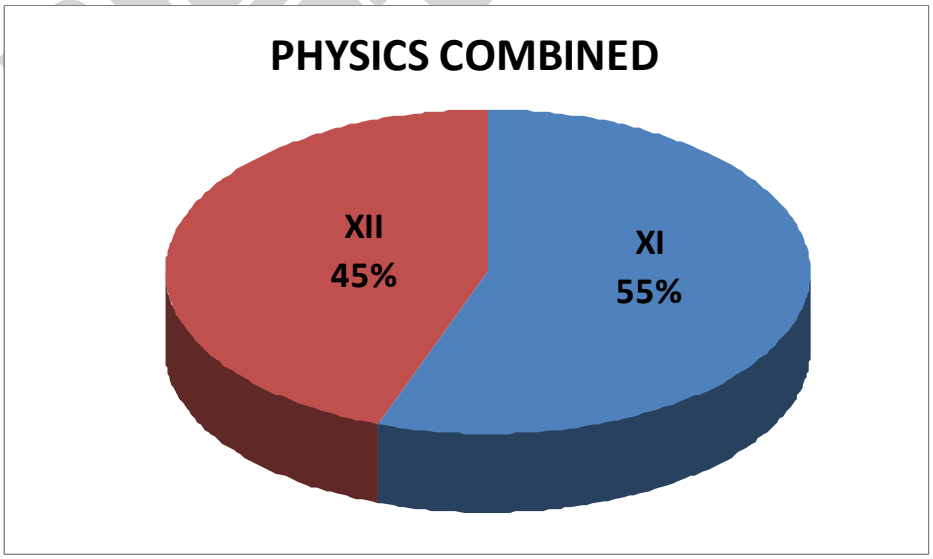
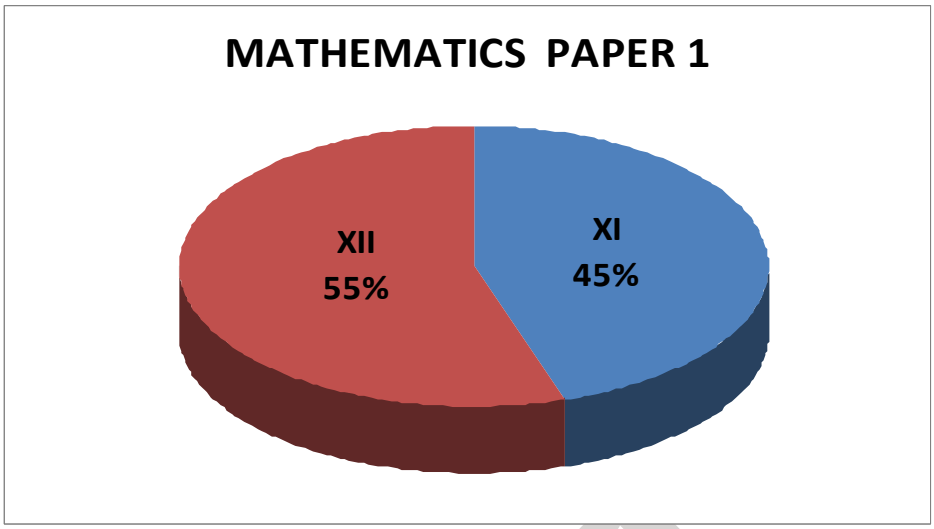
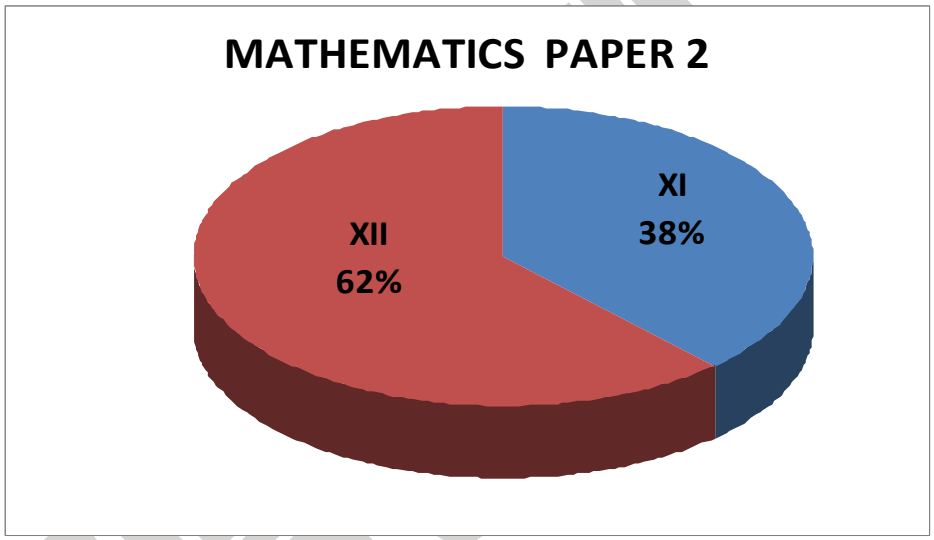


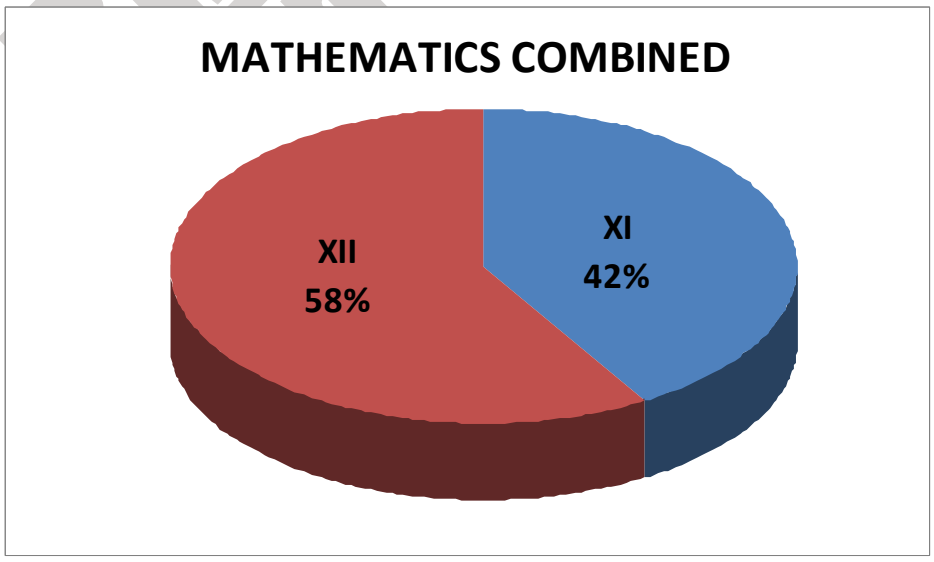




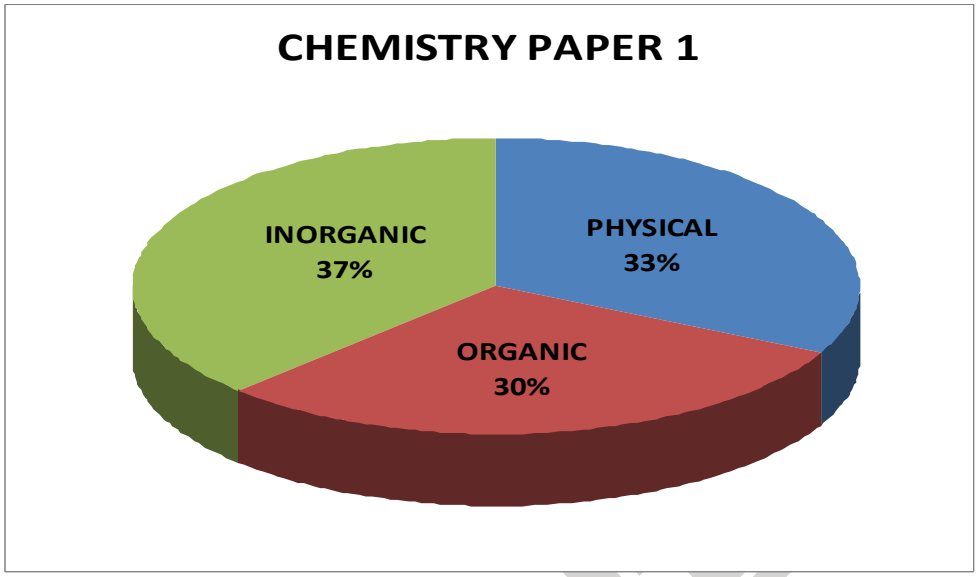
BREAK UP 2 (XI-XII)

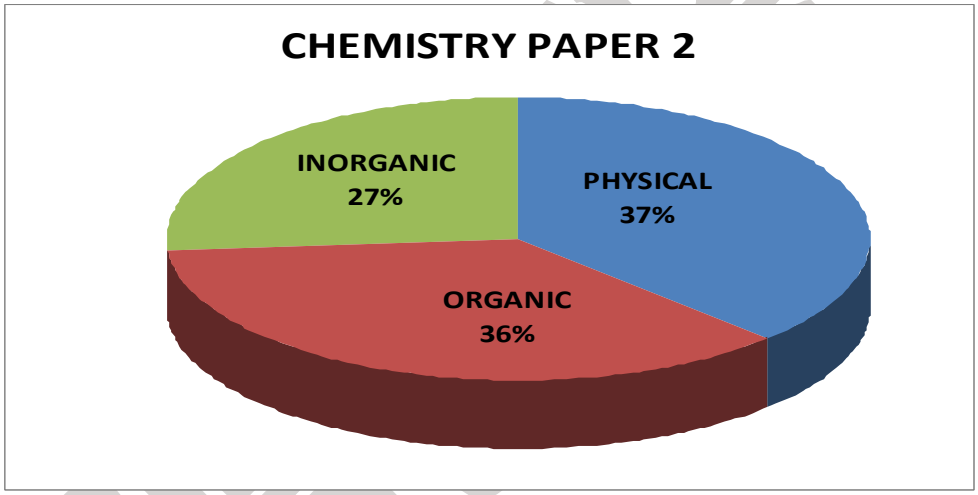


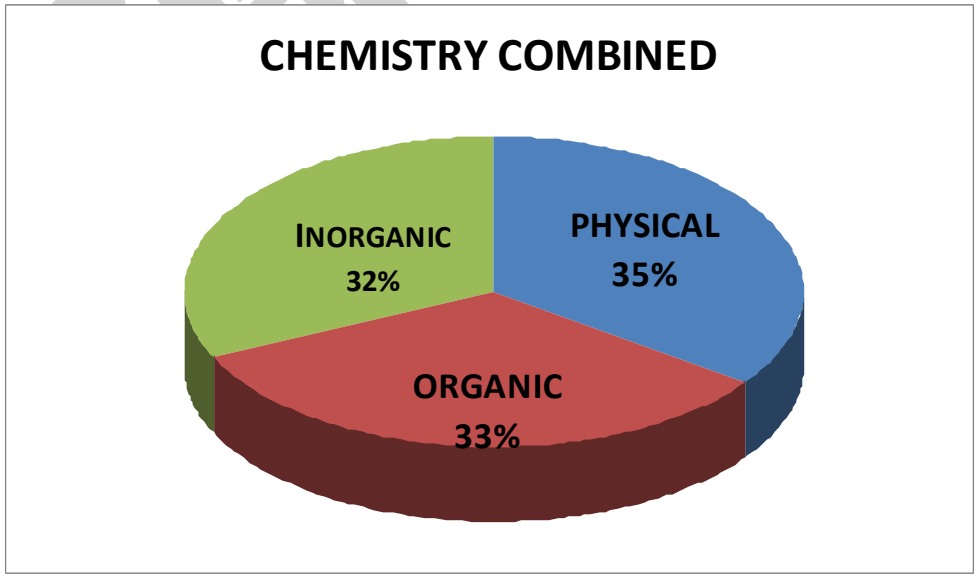




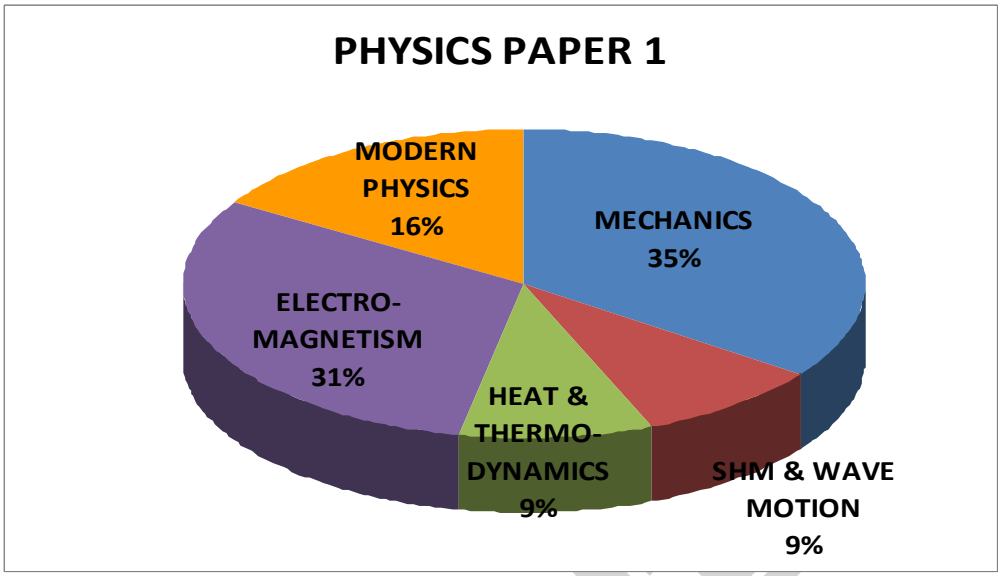
BREAK UP 3 (TOPICWISE/PARTWISE)

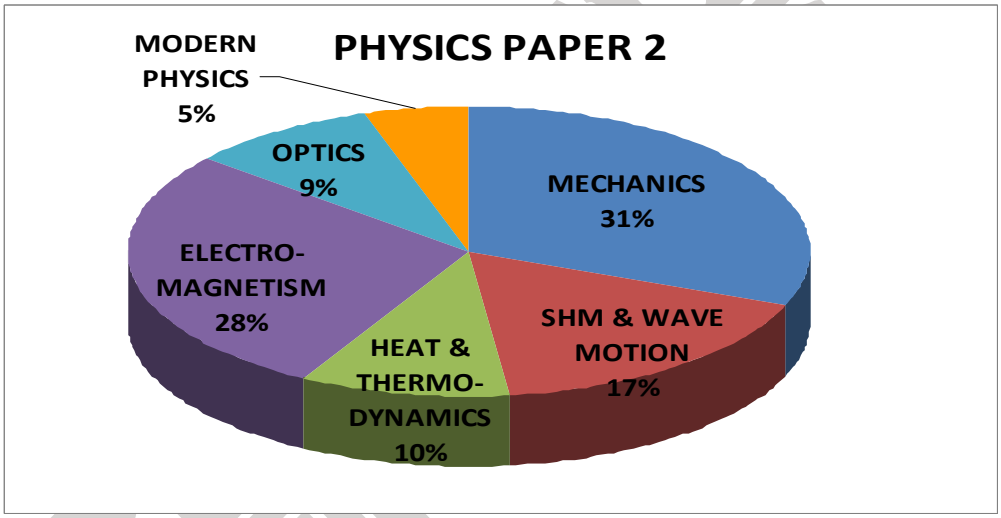


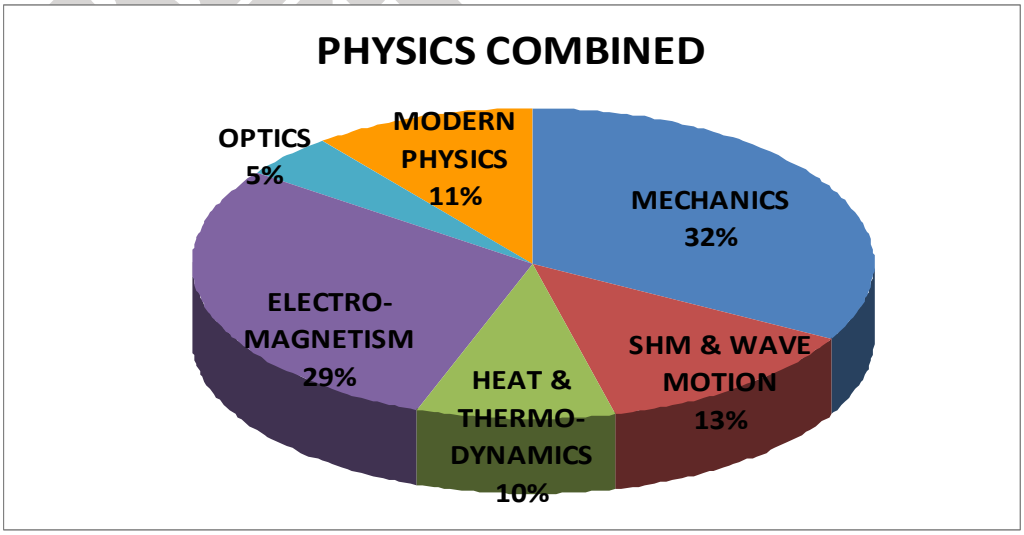




BREAK UP 3 (TOPICWISE/PARTWISE)







BREAK UP 3 (TOPICWISE/PARTWISE)